## 3-rd Mediterranean Mathematical Competition 2000

- 1. Let  $F = \{1, 2, ..., 100\}$  and let G be any 10-element subset of F. Prove that there exist two disjoint nonempty subsets S and T of G with the same sum of elements.
- 2. Suppose that in the exterior of a convex quadrilateral ABCD equilateral triangles XAB, YBC, ZCD, WDA with centroids  $S_1, S_2, S_3, S_4$  respectively are constructed. Prove that  $S_1S_3 \perp S_2S_4$  if and only if  $AC \perp BD$ .
- 3. Let  $c_1, \ldots, c_n, b_1, \ldots, b_n$   $(n \ge 2)$  be positive real numbers. Prove that the equation

$$\sum_{i=1}^{n} c_i \sqrt{x_i - b_i} = \frac{1}{2} \sum_{i=1}^{n} x_i$$

has a unique solution  $(x_1, ..., x_n)$  if and only if  $\sum_{i=1}^n c_i^2 = \sum_{i=1}^n b_i$ .

4. Let *P*, *Q*, *R*, *S* be the midpoints of the sides *BC*, *CD*, *DA*, *AB* of a convex quadrilateral, respectively. Prove that

$$4(AP^2 + BQ^2 + CR^2 + DS^2) \le 5(AB^2 + BC^2 + CD^2 + DA^2).$$