

**2007**

- 1** Let  $a, b, c$  be positive real numbers. Prove that

$$1 + \frac{3}{ab + bc + ca} \geq \frac{6}{a + b + c}.$$

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- 2** In a trapezoid  $ABCD$  with a base  $AD$ , point  $L$  is the orthogonal projection of  $C$  on  $AB$ , and  $K$  is the point on  $BC$  such that  $AK$  is perpendicular to  $AD$ . Let  $O$  be the circumcenter of triangle  $ACD$ . Suppose that the lines  $AK, CL$  and  $DO$  have a common point. Prove that  $ABCD$  is a parallelogram.
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- 3** Natural numbers  $a, b$  and  $c$  are pairwise distinct and satisfy

$$a|b + c + bc, b|c + a + ca, c|a + b + ab.$$

Prove that at least one of the numbers  $a, b, c$  is not prime.

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- 4** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that satisfy

$$f(x^3 + y^3) = x^2 f(x) + y f(y^2)$$

for all  $x, y \in \mathbb{R}$ .

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- 5** Let  $n$  be a natural number divisible by 4. Determine the number of bijections  $f$  on the set  $\{1, 2, \dots, n\}$  such that  $f(j) + f^{-1}(j) = n + 1$  for  $j = 1, \dots, n$ .
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