

**VI – IX**



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	5
1.	7
2.	9
3.	16
4.	19
5.	25
6.	29
7.	38
8.	41
9.	43
10.	46
11.	49
12.	52
1.	57
2.	63
3.	79
4.	88
5.	114
6.	128
7.	154
8.	163
9.	170
10.	179
11.	186
12.	195
	211







**1.**

1.  $A, B, C$  -  
 $A, B, C, C \cup (A \setminus B), C \cap (B \setminus A)$ .

2.  
 $A = \{n \in \mathbb{N} \mid n < 2021 \quad 7 \mid n\},$   
 $B = \{n \in \mathbb{N} \mid n < 2021 \quad 21 \mid n\},$   
 $C = \{n \in \mathbb{N} \mid n < 2021 \quad 84 \mid n\}.$   
 $\setminus (B \setminus C).$

3.  $A, B, C$   
 20 30.  
 $A \cap B = \{23, 28\},$   
 $A \cap C = \{28, 29\},$   
 $B \cap C = \{28\},$   
 $A \cup B = \{21, 22, 23, 24, 28, 29\},$   
 $A \cup C = \{21, 22, 23, 25, 26, 27, 28, 29\},$   
 $B \cup C = \{n \in \mathbb{N} \mid 23 \leq n < 30\}.$   
 $A, B, C.$

4.  $A, B, C$   $A \cap B \cap C = \emptyset.$  -  
 $A \setminus B = 7,$   $C \setminus B = 8,$   
 $A \cap C = 2$   $A \cup B \cup C = 20$   
 $B?$

5. 1000 -  
 700 : , 750 800 -

- 
6.  $94$   $40$   $20$   $27$   $60$   $4$   $?$
7.  $A = \{1, 2, 3, \dots, 100\}$ .  $A$ ,  $5035$ .
8.  $A = \{9, 10, \dots, 63, 64\}$   $2011$ .  $A$ .
9.  $A = \{\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}\}$   $1?$
10.  $A$   $\{1, 2, 3, \dots, 2015\}$   $675$   $A$ ,  $6$ .
11.  $(3, 3^2, 3^3, \dots, 3^{n-1}, 3^n)$   $n = 2022$ ,  $n = 2023$ .
12.  $M = \{1, 2, 3, \dots, 2023\}$ .  $M$   $?$

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2.

1.

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” “  
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( ) ，  
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2.

$10^{10}$  (  $10^{10}$  )，  
7  
23 ?

3.

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4.

5 -  
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5.

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:  
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16. 2022 2023 j .  
 A j B, 2021 B . ( A.)  
 j j .

17. 62. o 60.

18. n k  
 ,  
 :  
 1)  $n+1$  k,  
 2)  $n=2k+5$ ,  
 3)  $n+k$  3,  
 4)  $n+7k$  .

19. :  
 - ,  
 :  
 - ?  
 - , - , -  
 ,  
 - ? -  
 - , - -

20. 11 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ( )  
 .  
 „ k “, k  
 ? 11

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21. 1, 2, 3, 4 5 .  
, -  
.  
?

22. , .  
, , .  
, 3, 6, 15 18.  
, .

23. .  
-  
. , .  
, .  
?

24. .  
-  
. , .  
, .  
?

25. 100 kg .  
35 kg ,  
25 kg . ?

26. . 2 ,  
50 .  
-  
20  
5 . 95 ? ,

27.

143

?

28.

?

29.

55

1 55.

5

1.

1, 4, 7, 10, ..., 52 55

72

?

30.

21

1

21,

7

3

:

)

)

18

6

3

31.

*A, B, C, D, E, F, G, H, I, J*

2x5

?

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>

<i>G</i>	<i>A</i>	<i>E</i>	<i>F</i>	<i>B</i>
<i>I</i>	<i>C</i>	<i>J</i>	<i>D</i>	<i>H</i>

---

32.

2008

6, 7, 8, 9, ..., 2012, 2013.

33.

)

)

)

?

,

?

**3.**

1.  $25 \cdot 42 \cdot 50 \cdot 78 \cdot 105$  -

,  
?  
50

2.  $11 \cdot 10 \cdot 8$  -  
?  
8

3.  $100 \cdot 26 \cdot 100$  -

4.  $30^{100}$  -

5.  $5$  -  
?  
5

6.  $( \quad )$  -  
4 m  
7 m.  
1 m 4 m.  
) 16  
10 cm, 2  
m, 3 m 2 m

)  $2$   $17$   $?$   $?$   
 $2\text{ m}, 3\text{ m}$   $2\text{ m}$   $2$   
 $?$

7.

$A, B, C, D, E$   $M$   $A, B, C, D, E$   $M$   
 $\frac{n}{2}$ ,  $n$  ;  $M$   
 $M$   $M$

8.

$x$  ( $1\text{ cm}$ )  
 $1\text{ mm}$   $x$   $11$

9.

$1\text{ cm}$   $:$   $1\text{ cm}$   
 $1\text{ cm}$   $($

10.

$P = \{1, 2, 3, \dots, 100\}$  .  
 $51$   $5$

11.

$2\text{ cm}$   $193$   $4$   $\frac{\sqrt{3}}{4}\text{ cm}^2$  .

12.  $a, b, c, d$   $a > b > c > d$

$$S = (a-b)(a-c)(a-d)(b-c)(b-d)(c-d).$$

)  $3 \mid S.$

)  $4 \mid S.$

13. ) , 2015  
4030, 2015

2015.

) , 2016  
4030 ( ) 2015.

14. 6 .

( ) .



15.  $k$  ,  $k$  -  
4.

16. ,  
?

---

**4.**

1. -  
?

2.  $\{2, 4, 6, 8, 10, \dots\}$ .  
234. -  
?

3. 1 2013.  
3,  
11. ?

4. 22, 11,  
2. ?

5. 11- , 9,  
\*2013\*2013\*, ( )  
0, 1, 2 3?

6.  $n$   
100 11.

7.  $n$ , 100  
 $n^n$  .

8. 1000,  
9  
2 3.

9. 3 2 25  
.

10. -

11. -  
1, 2 3,  
X, 1, 2 3,  
X

12. 7, 8 9.

13. :  
) 1, 2, 3, 4, ) 0, 7, 8, 9, ) 1, 2, 3, 4, 5, 6?

14. 7, 8, 9 0.

15.

16. ,  
1, 2 3,  
1. 16-

17.

18. .( .)

19. 3720147410273 :  
1) 2014.  
2)

---

20. 3, 5, 7, 9, 1, ?

21. 1, 2, 3, 0, 1, 2, , 0,  
3 ?

22.  $(a+b)(c+d)(e+f) = 315$  -  
1 6 .  
?

23. 6.

24. 2023 ?

25.  $b = a + c$   $d = c + e$ .  $\overline{abcde}$

26. 5 , 1, 3  
? . , -

27. , -  
? .

28. 1, 2, 3, 4,  
?

29. ,  
?

30. 0, 2, 4, 5 6?

31. ) 10, ) 8? :

32. , 1, 2, 3, 4, 5 : - ) , ? )

33. 1, 2, 3, 4, 5 6 ( 5, ) , 1 2 ?

34. 8, 9 0, 6, 7, ?

35. 0, 1, 2 3. , 0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33, 100, ... 2013?

36. 2014? 7 -

37. {0,1,2,3,4,5} 5?

38. 5

39. 777, 0, 1, 2, 3 4.

40. 555 9999 0, 1, 3, 5, 7,

41. , 30000,  
2, 3, 4, 5, 3, 4, 5  
, 2 ?

42. 816239745 :  
) 1 9 ,  
) 6, 7, 8, 9 12345,  
) 7, 8, 9 123456.  
, -  
) , ) ).

43. 4 , -  
.

44.  $\overline{abcd}$  4,  $\overline{abcd}$   
 $\overline{abc}$   $\overline{bcd}$   
3.

45. :  
) 330, ) 450.

46.  $N = 2^8 \cdot 3^8 \cdot 5^8 \cdot 7^8 \cdot 11^8 \cdot 13^8 \cdot 17^8 \cdot 19^8$   
8 .

47. :  
- 30,  
- 3  
- 300 .

48.  $(a, b, c)$  -  
,  $abc = 9000$ .

49.  $(a, b, c, d)$   
 $abcd = 15000$ .

50.

$$(a,b,c) \quad a^4b^2c = 54000?$$

51.

$$\frac{2}{1-3} \quad 3. \quad ?$$

1,

52.

$$\frac{p}{q} \quad (0,1) \quad M(p,q) = pq$$

$$M(p,q) = 25!.$$

53.

$$5, 6 \quad 7, \quad 5, 6$$

54.

3?

55.

56.

$$n \geq 3 \quad A \quad n-10 \quad B$$

57.

$$n \geq 3 \quad A \quad 3, \quad 10, \quad n-5 \quad 7. \quad B > A.$$

58.

$$a_1, a_2, \dots, a_{10} \quad 1, 2, 3, \dots, 10$$
$$a_1 + a_2 + \dots + a_{10} \quad 3. \quad a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots,$$

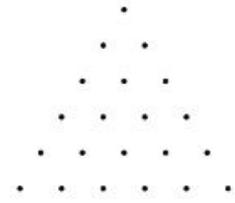
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**5.**

1.

?

2.



3.

?

4.

)  
)

6

?

5.

$p$

$p \cdot$

?

6.

8

?

7.

$AC \quad BC$

$ABC$

9

?

8.

$AC \quad BC$

$ABC$

$n$

?

9.

5

10.

2 cm.

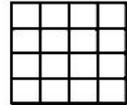
1 cm

11.

1 cm,

4 cm

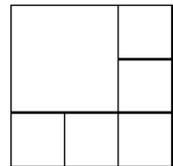
16



)  
)

?

12.



13.

16

4

14

?

14.

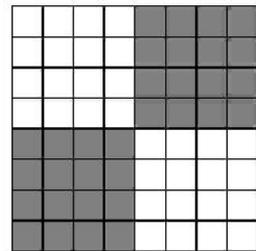
( )

8x8

2x2, 4x4

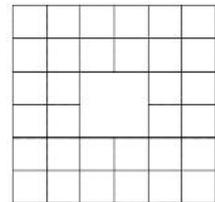
6x6,

?



15.

( )

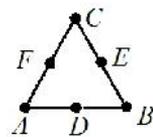


16.

D, E F

ABC,

A, B, C, D, E F.



17.

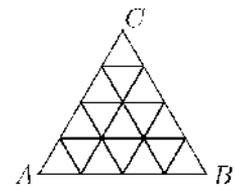
$ABC$

16

(  
-  
-)

).  
 $87 \text{ cm}^2$ .

$\triangle ABC$ .



18.

,

2009,

-

,

-

2012,

-

19.

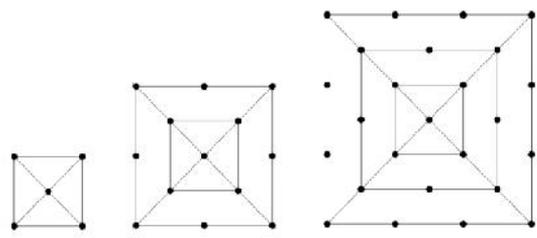
5

,

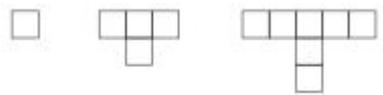
13

.

,  
?



20.



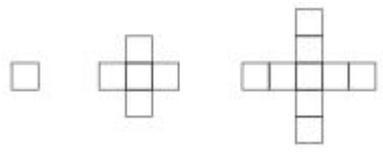
?

?

$n -$

?

21.



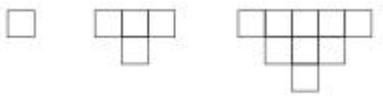
?

?

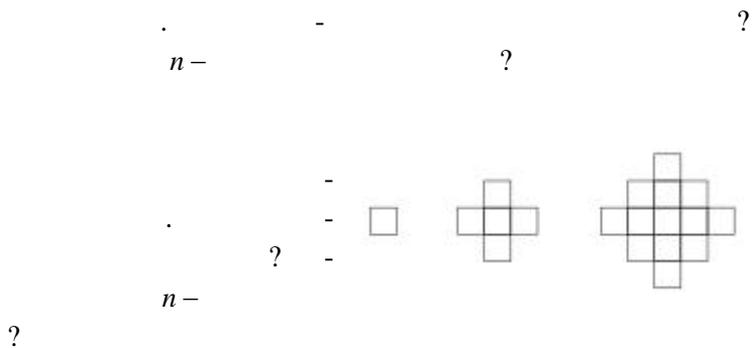
$n -$

?

22.



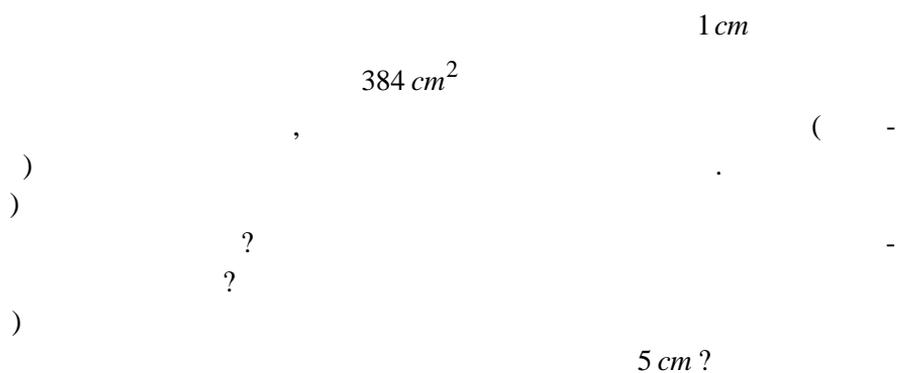
23.



24.



25.



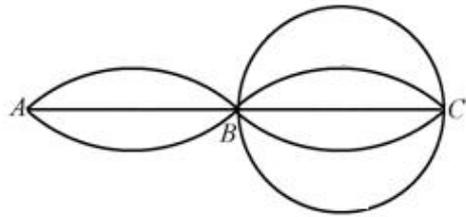
6.

1. ( ): , ):

$$\begin{array}{r} ABBCB \\ +BCADA \\ \hline DBDDD \end{array}$$

2. 1, 2, 3, 4, 5 6 . - , . ?

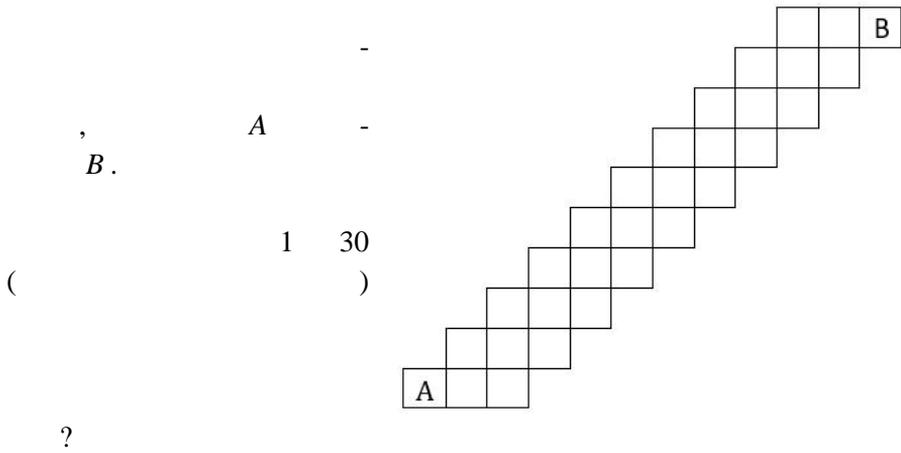
3. A, B C -  
-  
-  
A C -  
,  
A C  
B .  
A B ,  
B C .  
A C ?



4. A B  
4 , B C  
3 . A  
B C .

5. 8 . -  
,  
, ?

6.



7.

10

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8.

9.

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10.

2023

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11.

2023

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12. 3 5 -  
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13. , , , , .  
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14. 5  
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15. :  
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16. : -  
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17. .  
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18. 6  
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19. 25 . -  
 , -  
 .
20. 10 5 5 ,  
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-

?

21.

1 9. :  
) 1 2 ,  
) 1 2 ?

22.

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) , ? ,  
) ? ,  
) ?

23.

12 . -  
, -  
? .

24.

, , ( ,  
) . , -  
? . -

25.

, - ,  
, -  
? , -

26.

, , 1 3  
, 2 , 2

---

27. 7 5 . ?

28. , 3 2 ?  
, 50 , 1 2 . :5 , 10 , 20

29. 85 ? -

30. ? -

31. , , , . , ?

32. , 300 .

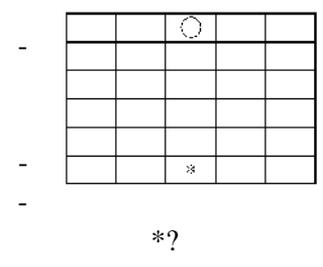
33. ( ) ?

33. 36, - ,



42.

○  
\*.  
,  
.  
○



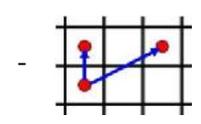
43.

8  
1, 2, 3, 4, 5, 6, 7, 8.

. , -  
. , -  
. , -  
1 8 ,  
.

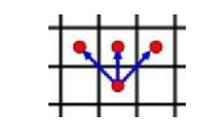
44.

9×9.  
,  
?



45.

7×7 .  
- ( ).  
- ?



46.

5 , -  
?

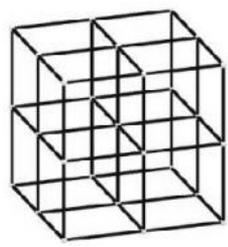
47.

xyz 3×1

$x \leq 3, y \leq 3, z \leq 3$  ( -  
 $x, y, z$  ).  $xyz$  -  
 $zyx$  .

)  
 )  
 ,  $abc$   
 $bcd$  (  $a, b, c, d$  ).

48.  $1 \times 1 \times 1$  12 -  
 $2 \times 2 \times 2$  42 -  
 ( ) .  $8 \times 8 \times 8$  -  
 $9 \times 9 \times 9$  .

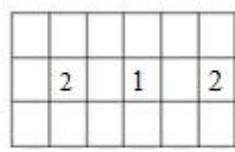


49. 10 ,  
 ? ,

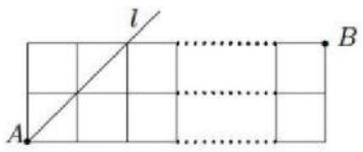
50. (0,0) -  
 1 . -  
 (6,14)?



51. -  
 , -  
 . -  
 ( ) .  
 ?



52.  $2 \times 2011$  ( -  
 ) .



- 
- $A$        $B$ ,
- $l$                                    $A$ .
53.                                  .                                   $(0,0)$
- $(5,1)$
- $S = \{(i, j) \mid i = 1, 2 \quad j = 1, 2, 3, 4, 5\}$  .
- $S$                                    $S$
- .                                  .
- $(0,0)$                                    $(5,1)$
- ? (
- .)
54.                                  12      .
- ,
- .                                  ?
55.                                  2010                                   $a, b, c$
- .

7.

1.

45 .

?, ( 0  
0,5 .)

-  
-  
1



2.

130 .

?

3.

, , , ,  
- .  
12 , 6 .  
?

4.

5 ,

20 ,  
2 .  
?  
( 3 , .)

5.

7 ,

53 . 3 , 0 ? ( .)

6.

5 ,

7. ( 7, 6, 5, 4, 3, 2, 1, 0 ) ?

7. 15 -  
 30 1, 0, 3, 0 -  
 ?

8. 10 -  
 3, 0, 1, M, m -  
 ) M ? 98,  
 ) M ? 98  
 M = 19, m ?

9. 3, 2, 0, 1, 3:0, 3:1, 3:2  
 A, B, C, D.

A	9	9	1
B	5	7	6
C	2	4	8
D	2	4	9

10.

$a, b, c$  are positive integers such that  $a > b > c$ .  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.

11.

The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.  
The numbers  $a, b, c$  are in arithmetic progression.  
The numbers  $a, b, c$  are in geometric progression.  
The numbers  $a, b, c$  are in harmonic progression.

**8.**

1.  $A = \{5, 6, 7, \dots, 74, 75\}$   
 :  $A$ ,  $A$ , ?

2. 90 -  
 . 2023  
 ,  
 .

3.  $n \geq 3, n \in \mathbb{N}$ .  
 $n^3$  1.  
 $A$ ,  $B$ ,  $C$   
 $D$   
 $A = B, A = C, A = D, B = C, B = D, C = D$   
 $n$ , .

4.  $n \geq 3$ .  $1, 2, \dots, n$ .  
 10 , -  
 .  
 $n$  ).  $n$  ( 1

5. , -  
 ,  $2 \times 3$  (  $3 \times 2$  )  
 6 , 2  
 $9 \times 11$  ?

6. 16 .  
 ,

7.  $18 \times 18$   $18 \cdot 18 = 324$

8.  $5 \times 5$



15	11	37	11	15
11	21	40	21	11
37	40	26	40	37
11	21	40	21	11
15	11	37	11	15

9.  $2 \times 2$   $7 \times 7$

10.  $10$   $10$   $1.$   $k$   $k$

11.  $25$   $125$   $($   $)$   $($   $)$

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**9.**

1.  $\dots$  ( ),  
100  $\dots$  -

2.  $x$  ( ) , 10 , 20 , 40 , 80 -  
?  $x$  -

3. 16  $\dots$   
:  
,  
,  
.

4. 1, 2, 3, ..., 20. -  
?  
?

5.  $5 \times 5$  ,  
X O  
,  
( )  
X, -

6.  $100 \cdot 100 = 10000$  (10000) ? ( )

7.  $2023 \times 2023 = 4092529$  (4092529) ? ( )

8.  $1000000 \cdot 1000000 = 1000000000000$  (1000000000000) ? ( )

9.  $1000000000000 \cdot 1000000000000 = 1000000000000000000000$  (1000000000000000000000) ? ( )

$s_p$				$m_p$
<b>M</b>				<b>S</b>

10.  $1000000000000000000000 \cdot 1000000000000000000000 = 100$  (100) ? ( )

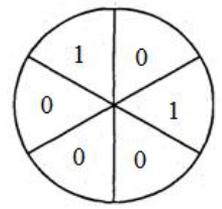
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, , -  
3, , .  
3 . ?  
9. 2013 -  
11 .  
( ).  
, ?

10.

1.  $\overline{abcd}$  :  $\overline{abcd}$   
 0, 1, 9, -  
 1.  $\overline{abcd}$  :  
 2621 ) 2016, ) 2128.

2. 6 , 1, 0, 1, 0, 0, 0. 1



3. 1, 2, 3, 4, 5, 2022. -  
 $a-1$   $a+1$ ,  $a$   $b$ ,  $a$   $b-3$   $b+3$ . -  
 : 2023, 5, 4, 3, 2, 1.

4.  $(a,b)$  : I  
 $(a-b,b)$ , II  $(a+b,b)$   
 III  $(b,a)$ . -  
 (19,94). , -  
 :  
 ) (19,96), ) (19,95).

5. 2015 :

1000

1004

153?

1, 5 3,

6.

25

5 × 5

1

:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

?

7.

8 × 8

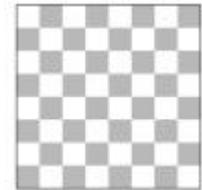
, . . .

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-

-



?

8.

10 cm ,

100 -

1 cm .

25

4 cm

1 cm .

?

9.

$m \times n$

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-  
.

10.  $2n (n > 1)$  . ,  $n$   
,

11. .  
,  
( .)



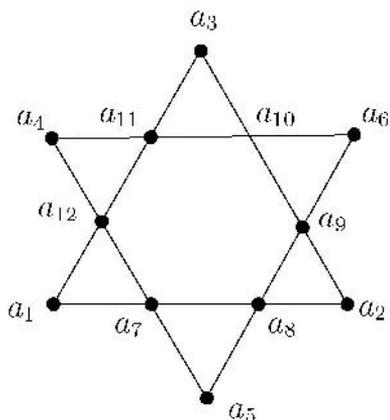
( - )  
 ( - ),  
 ) ( ) . ( -

5.

$a_1, a_2, \dots, a_{12}$

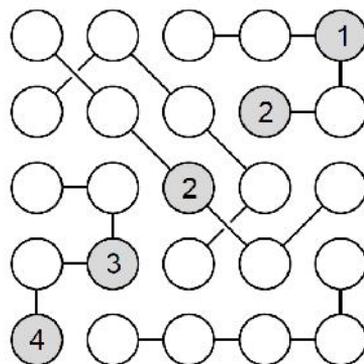
1 12

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6.$$



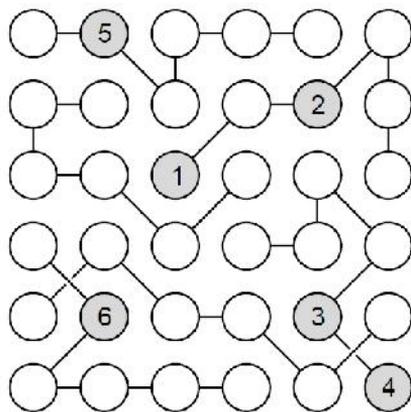
6.

1, 2, 3, 4 5.



7.

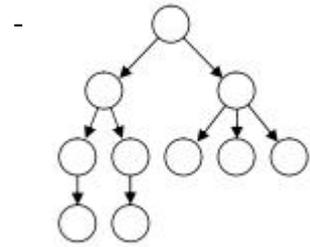
1, 2, 3, 4, 5 6.



8.

1, 2, 3, ..., 10

(  
)



9.

9

(  
)

3x3

?

10.

2013

,

,

?

11.

3x3x3,

1 27

,

-

?

,

?



12.

1.  $12\text{ cm}$ ,  
 $15\text{ cm}$ .  
 ?

2.  $1$ ,  $\frac{1}{2}, \frac{1}{3}$   
 $\frac{1}{4}$ ,  
 ?

3.  $23$ ,  $15$ ,  $24$   
 ?

4. ( )

„broj“	1	2	3	4	5	6	7	8	9	0
○ ●	● ○	● ○	● ●	● ●	● ○	● ●	● ●	● ○	○ ●	○ ●
○ ●	○ ○	● ○	○ ○	○ ●	○ ●	● ○	● ●	● ●	● ○	● ●
● ●	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○

$11 =$  ○ ● ● ○ ● ○

$478 =$  ○ ● ● ● ● ○ ○ ○

” “

13 -

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5.

27,

22.

?

6.

$n$

$n$

7.

15

?

8.

$n < 2013$

$n$ .

9.

14.

?

10.

$A \quad B$

$A$   
 $B$

1 15

,  $B$

$A?$

11.  $a' b'$ ,  $a'' b''$

12.

13.  $21$ ,  $10$ ,  $20$ ,  $I$

14.  $44$ ,  $44$ ,  $?$  ( )

15.  $41$ ,  $41$ ,  $42$ ,  $42:2=21$ ,  $42:7=6$ ,  $42:3=14$



,

?

16.

0	1	2	3	4	5	6	7	8	9
9	0	1	2	3	4	5	6	7	8
8	9	0	1	2	3	4	5	6	7
7	8	9	0	1	2	3	4	5	6
6	7	8	9	0	1	2	3	4	5
5	6	7	8	9	0	1	2	3	4
4	5	6	7	8	9	0	1	2	3
3	4	5	6	7	8	9	0	1	2
2	3	4	5	6	7	8	9	0	1
1	2	3	4	5	6	7	8	9	0

,

17.

8×8

11, 12, 13, 14, 15, 16, 17 18,  
21, 22, 23, 24, 25, 26, 27

28,  
86, 87 88. 8

81, 82, 83, 84, 85,

81	82	83	84	85	86	87	88
71	72	73	74	75	76	77	78
61	62	63	64	65	66	67	68
51	52	53	54	55	56	57	58
41	42	43	44	45	46	47	48
31	32	33	34	35	36	37	38
21	22	23	24	25	26	27	28
11	12	13	14	15	16	17	18

,

18.

8×8

1, 3, 5, 7, 9, 11, 13, 15;  
17, 19, 21, 23, 25, 27, 29, 31  
1, 19 3 . ( );  
33, 35, 37, 39, 41, 43, 45, 47

17

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55				

8

?

- 
19.  $\frac{a}{b}$ ,  $a$ ,  $b$  -  
 $\frac{a}{b}$  1
20.  $n$ ,  $a_1, a_2, \dots, a_n$  -  
 $i = 1, 2, \dots, n$   $a_i \leq a_{i+1} \leq 2a_i$ .  $a_1 = 1$ ?
21.  $m$  -  
 $m$  3.
22.  $n - 2012$  -  
 $n$  3.
- 23.
24. 40,  $(0, 1)$  -  
 1
25.  $n$ ,  $( \quad )$  -  
 $n$

**1.**

1.  $A, B \subseteq C$  -

$$A, B, C, C \cup (A \setminus B), C \cap (B \setminus A).$$

$$A = \{1, 2, 3, 4, 5, 6, 7\}, B = \{2, 3, 4, 5, 6, 7\}$$

$$C = \{1, 2, 3, 4, 5, 6, 7\}. A \setminus B = \{1\} \subseteq C \quad B \setminus A = \emptyset,$$

$$C \cup (A \setminus B) = C \quad C \cap (B \setminus A) = \emptyset.$$

2.

$$A = \{n \in \mathbb{N} \mid n < 2021 \wedge 7 \mid n\},$$

$$B = \{n \in \mathbb{N} \mid n < 2021 \wedge 21 \mid n\},$$

$$C = \{n \in \mathbb{N} \mid n < 2021 \wedge 84 \mid n\}.$$

$$\setminus (B \setminus C).$$

$$A = \{7, 14, 21, \dots, 2016\} \quad 288 \quad -$$

$$B = \{21, 42, 63, \dots, 2016\} \quad 96 \quad ,$$

$$C = \{84, 168, 252, \dots, 2016\} \quad 24 \quad . \quad C \subset B,$$

$$B \setminus C \quad 96 - 24 = 72 \quad , \quad B \setminus C \subset A$$

$$\setminus (B \setminus C) \quad 288 - 72 = 216 \quad .$$

3.  $A, B \subseteq C$

$$20 \quad 30.$$

$$A \cap B = \{23, 28\},$$

$$A \cap C = \{28, 29\},$$

$$B \cap C = \{28\},$$

$$A \cup B = \{21, 22, 23, 24, 28, 29\},$$

$$A \cup C = \{21, 22, 23, 25, 26, 27, 28, 29\},$$

$$B \cup C = \{n \in \mathbb{N} \mid 23 \leq n < 30\}.$$

$$A, B \subseteq C.$$

$$B \cup C = \{23, 24, 25, 26, 27, 28, 29\}.$$

$$A \cap B = \{23, 28\}, A \cap C = \{28, 29\} \quad B \cap C = \{28\}$$

$A \cap B \cap C = \{28\}.$

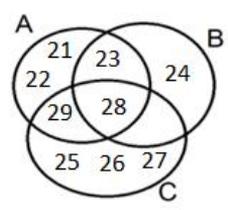
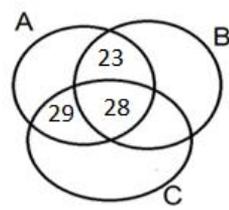
( 21 22 ) .

$B \cap C = \{24, 25, 26, 27\}$  -

$A \cap C = \{28, 29\}$  -

$A \cap B = \{23\}$  .

( ) :



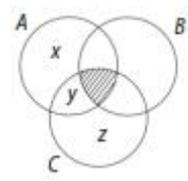
$A = \{21, 22, 23, 28, 29\}, B = \{23, 24, 28\}, C = \{25, 26, 27, 28, 29\}.$

4.  $A, B, C$   $A \cap B \cap C = \emptyset.$  -

$A \setminus B = 7,$   
 $A \cap C = 2$

$C \setminus B = 8,$   
 $A \cup B \cup C = 20$

$B?$



-  $y = 2,$   $A \cap B \cap C = \emptyset,$   $A \cap C = 2$  -

-  $x + y = 7,$   $A \setminus B = 7,$   $x = 5$

-  $y + z = 8,$   $C \setminus B = 8,$   $z = 6.$

$A \cup B \cup C = 20,$   $B = 20 - (x + y + z) = 20 - 13 = 7.$

5.  $1000$  -

$700$   $750$   $800$  -

?

$700 + 750 + 800 = 2250$

$x$

,  $y$

$z$

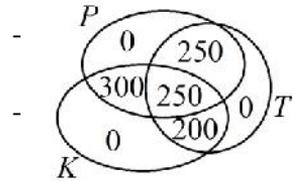
$$x + y + z = 1000 \quad 3x + 2y + z = 2250,$$

$$2250 = 3x + 2y + z = (x + y + z) + 2x + y = 1000 + 2x + y,$$

$$\therefore 2x + y = 1250. \quad , \quad x + y = 1000 - z \leq 1000,$$

$$x = 1250 - (x + y) \geq 1250 - 1000 = 250.$$

, 250



6.

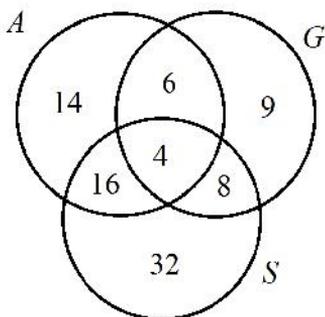
94

, 27  
60

40  
20

4

?



$$12 - 4 = 8$$

$$20 - 4 = 16$$
$$10 - 4 = 6$$

$$40 - 16 - 4 - 6 = 14$$
$$27 - 8 - 4 - 6 = 9$$

$$60 - 16 - 4 - 8 = 32 \quad (\quad).$$

$$14 + 9 + 32 = 55$$

$$94 - (14 + 6 + 9 + 16 + 4 + 8 + 32) = 5$$

7.  $A = \{1, 2, 3, \dots, 100\}.$

$A,$   $5035.$

$A :$

$$1 + 2 + \dots + 99 + 100 = (1 + 100) + (2 + 99) + \dots + (50 + 51) \\ = 50 \cdot 101 = 5050.$$

$A$

$$5050 - 5035 = 15.$$

:

{15},

{1,14}, {2,13}, {3,12}, {4,11}, {5,10}, {6,9}, {7,8},

{12,1,2}, {11,1,3}, {10,1,4}, {10,2,3}, {9,1,5}, {9,2,4}, {8,1,6}, {8,2,5}, {8,3,4},

{7,2,6}, {7,3,5}, {6,5,4},

{9,1,2,3}, {8,1,2,4}, {7,1,2,5}, {7,1,3,4}, {6,1,3,5}, {6,2,3,4},

{5,1,2,3,4}.

$$15 \quad 27$$

$$5035 \quad 27.$$

8.  $A = \{9, 10, \dots, 63, 64\}$

2011.

$A.$

$A$

$$1 + 2 + 3 + \dots + 63 + 64 - (1 + 2 + 3 + \dots + 8) = \frac{64 \cdot 65}{2} - \frac{8 \cdot 9}{2} = 2044.$$

33

:

{33},

:

{8,25}, {9,24}, {10,23}, {11,22}, {12,21}, {13,20}, {14,19}, {15,18}, {16,17}

:

{8,9,16}, {8,10,15}, {8,11,14}, {8,12,13}, {9,10,14}, {9,11,13}, {10,11,12}.

2011,

$$1 + 7 + 9 = 17$$

2011.

9.

$$A = \left\{ \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11} \right\}$$

1?

), 1. :  $\left(\frac{1}{11}, \frac{10}{11}\right), \left(\frac{2}{11}, \frac{9}{11}\right), \left(\frac{3}{11}, \frac{8}{11}\right), \left(\frac{4}{11}, \frac{7}{11}\right), \left(\frac{5}{11}, \frac{6}{11}\right)$ .  
 8, 6  
 4, 2, -  
 . , 8 + 6 + 4 + 2 = 20 -  
 .

10.

A {1, 2, 3, ..., 2015} 675  
 . A,  
 6.

. A 6,  
 A 6, 3. X  
 $336 + 336 = 672$  {1, 2, 3, ..., 2015} 6  
 1 5, Y  $336 + 336 = 672$   
 {1, 2, 3, ..., 2015} 6 2 4. A X  
 336,  
 6 (1 5),  
 6. , A X 336 . -  
 A Y 336 .  
 , A  $2 + 336 + 336 = 674$  ,  
 .

11.

$$3, 3^2, 3^3, \dots, 3^{n-1}, 3^n$$

( )

, :

)  $n = 2022$ ,

)  $n = 2023$ .

.

$n$

$$3 \cdot 3^2 \cdot 3^3 \cdot \dots \cdot 3^{n-1} \cdot 3^n = 3^{1+2+3+\dots+n} = 3^{\frac{n(n+1)}{2}}$$

$3 \mid \frac{n(n+1)}{2}$ .  
 $n = 2023$   
 $n = 2022$   
 $1, 2, 3, 4,$   
 $\dots, 2021, 2022$   
 $\vdots$   
 $\{1, 2, 3, 4, 5, 6\}, \{7, 8, 9, 10, 11, 12\}, \dots, \{2017, 2018, 2019, 2020, 2021, 2022\}$ .

12.

$$M = \{1, 2, 3, \dots, 2023\}.$$

$M$

?

$M$

$$\{a, b, a+b\}.$$

$$1 \leq a < b < a+b \leq 2023.$$

$$2a < a+b \leq 2023,$$

$$a \leq 1011,$$

$$a \in \{1, 2, \dots, 1010, 1011\}.$$

$a$

$b$

$$a+1, a+2, \dots, 2023-a.$$

$a$

$$2023 - a - (a+1) + 1 = 2023 - 2a$$

$b$ .

$$\begin{aligned}
 & (2023 - 2 \cdot 1) + (2023 - 2 \cdot 2) + (2023 - 2 \cdot 3) + \dots + (2023 - 2 \cdot 1011) = \\
 & = 1011 \cdot 2023 - 2 \cdot (1 + 2 + 3 + \dots + 1011) \\
 & = 1011 \cdot 2023 - 1011 \cdot 1012 \\
 & = 1011^2.
 \end{aligned}$$

2.

1.

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 . ( ) ,  
 250 ,  
 $6 \cdot 250 = 1500$  . 9  
 ,  $1500 = 166 \cdot 9 + 6$  ,  
 166 6- -  
 , . . ( ) .



2.

10 (   
 10 ) ,  
 7 .  
 23 ?  
 . -  
 , 7 ,  
 , 3  
 . 10  
 ,  $3 + 2 \cdot 10 = 23$  .

3.

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 : , , .  
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3, 4 5,  
 1 2. ,  
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 6. ( ). ,  
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 ( ), ( ).  
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 7. :  
 42.  
 7. ,  
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 ?  
 42 7 : 7, 14, 21, 28  
 35. 42 : 7, 7 28; 7, 14  
 21; 14, 14 14.  
 ,

7, 14 21 .

8. , , 4, 5, 6 7 . , , .

1) , :

2) ,

3) ,

4) 2

5) , 1 .

. 1), 2) 3) , .

7 . , 5 6

, 4 . , 2

, 5-2=3, 4 5 . 6 ,

, 4 . 7 ,

5 1 ,

5 . , 7

9. : , : „

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	+	-	-
	-	+	-

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	-			
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	-	-		
	+	-	-	-
	-	-		

	-	+	-	-
	-	-	+	-
	+	-	-	-
	-	-	-	+

11.



?

12.

$(D_1, D_2, D_3, M_1, M_2, M_3)$   
 $(D_1, M_1)$   
 $(D_2, M_2)$   
 $(D_3, M_3)$   
 $(D_1, M_2)$   
 $(D_2, M_3)$   
 $(D_3, M_1)$   
 $(D_1, M_3)$   
 $(D_2, M_1)$   
 $(D_3, M_2)$

13.

2023  
 2015  
 $\frac{2023-5}{2}$   
 $\frac{2023-5}{2}$

14.

146  
 31  
 $146 + 31 = 177$   
 $(177 + 1) : 2 = 89$

$$89 \cdot 89 = 7291$$

15. 13 2, 3, 4 5,

2, 3, 4, 5

3

12

$$3 \cdot (2 + 3 + 4 + 5) = 42,$$

13

$$: 42 + 2 = 44, 42 + 3 + 45, 42 + 4 = 46$$

$$42 + 5 = 47.$$

$$44, 45, 46 47$$

13,

16. 2022 2023 j

A j B, 2021 B A.)

j j

j

$$2023 \cdot 2021,$$

2021

j

j

j

d,

j

$$2022d,$$

$$2023 \cdot 2021 = 2022d,$$

( ,

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).

17.

62. o

60.

60,

5, 4 3.

5.

5 10,

$$1 + 2 + 3 + 4 + 6 + 7 + 8 + 9 + 11 + 12 = 63 > 62,$$

5.

4. 4, 8 12,

$$1+2+3+5+6+7+9+10+11+13=67 > 62,$$

4.

3. 3, 6, 9 12,

$$1+2+4+5+7+8+10+11+13+14=75 > 62,$$

3.

3, 4 5,

60.

18.

$n$   $k$

1)  $n+1$   $k$ ,

2)  $n=2k+5$ ,

3)  $n+k$  3,

4)  $n+7k$  .

3)  $n+k=3a$  .  
 $n+7k=3a+6k=3(a+2k)$  , -

4)  $3a-k=n=2k+5$  ,  $\dots 5=3(a-k)$  ,

2)  $n+k=3a$  , 3)

3.

1) 2)  $n+1=2k+6$   $k$  ,  
 $k|6$  ,  $k \in \{1, 2, 3, 6\}$  .

$k=1$   $n=7$  ,  $n+7k=14$  4) .

$k=2$   $n=9$  ,  $n+7k=23$  4) .

$k=3$   $n=11$  ,  $n+7k=32$  4) .

$k=6$   $n=17$  ,  $n+7k=59$  4) .

:  $k=2$  ,  $n=9$   $k=6$  ,  $n=17$  .

19.

:

- , :  
 - ?  
 - , - , -  
 - , ? -  
 - , - -  
 .  
 .  
 .  $a$  ,  
 $b$   $c$  -

$$a + c = \frac{1}{2}.$$

$$a \quad b. \quad b, \quad a + c + 2b = 1, \quad b = \frac{1}{4}.$$

$$2b = \frac{1}{2}, \quad \dots \quad a \geq \frac{1}{2} \quad c \geq \frac{1}{2}.$$

$$a + c = \frac{1}{2}.$$

$$2a + 2c = 1, \quad a. \quad 2a + b + c = 1$$

$$b = c.$$

$$a = 2(b + c) = 4b. \quad a + b = \frac{1}{2}$$

$$b = \frac{1}{10}, \quad \frac{2}{5},$$

$$\frac{1}{10}$$

20. 11 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ( ) .

„  $k$  “,  $k$  :  
 . 11

$$? \quad k \geq 6$$

$$, \quad 6$$

$$, \quad 11 - 6 = 5$$

$$, \quad k \geq 6$$

$$6$$

6

0, 10, 1, 9, 2, 8, 3, 7, 4, 6, 5.

6

21.

1, 2, 3, 4, 5

?

12

: (1,2,3,4,5) → (1,6,3,4,1) → (1,0,9,4,1) → (1,0,1,12,1).

3.

15

12,

15

15 (

5

10

10

3,

10

10

22.

3, 6, 15, 18.

$$3 + 6 + 15 + 18 = 42$$

42.

23.

?

$$15 - 6 = 9$$

$$32 - 5 = 27$$

$$9 + 27 = 36$$

24.

?

$$18 - 8 = 10$$

$$28 - 8 = 20$$

$$10 + 20 = 30$$

$$28 + 2 = 30$$

25.

100 kg .

35 kg ,

25 kg .

?

35 kg ,

11 kg (

$$3 \cdot 12 = 36 \text{ kg} > 35 \text{ kg} ).$$

25 kg ,

9 kg (

$$3 \cdot 8 = 24 \text{ kg} < 25 \text{ kg} ).$$

9 11 kg .

$$100 - (35 + 25) = 40 \text{ kg} .$$

$$\frac{40}{11} = 3\frac{7}{11} \quad \frac{40}{9} = 4\frac{4}{9} ,$$

4

$$3 + 3 + 4 = 10$$

26.

$$\begin{aligned}
 & 20 + 5 + 95 = 120 \\
 & 2n + 0,5(n-1) + 0,2(2n-2) + 0,05(4n-4) = 3,1n - 1,1 \\
 & 95 = 3,1n - 1,1 \\
 & 3,1n - 1,1 = 95 \\
 & 3,1n = 96,1 \\
 & n = 96,1 : 3,1 \\
 & n = 31 \\
 & 8 \cdot 31 - 7 = 241
 \end{aligned}$$

27.

$$\begin{aligned}
 & 143 = 11 \cdot 13 = 1 \cdot 143 \\
 & 143 = 11 \cdot 13 \\
 & 11 \cdot 10 = 110 \\
 & 143 + 110 = 253 \\
 & 13 \cdot 12 = 156 \\
 & 143 = 11 \cdot 13
 \end{aligned}$$

28. 11 23

$n-1$   
 $n-1$   
 $n-1=42$ ,  $n=43$ .  
 $NZS(2,3,7)=42$ ,  $n-1=42$ ,  $n=43$ .  
 $22$   
 $15$   
 $7$   
 $22+15+7=44$   
 $84$ ,  $n=85$ .  
 $n-1$   
 $43$   
 $13$   
 $29$

29. 55

$1$  55.  
 $1$ .  
 $1, 4, 7, 10, \dots, 52$  55  
 $72$   
 $?$   
 $4+5=9$   
 $19-$   
 $36$   
 $1, 4, 7, 10, \dots, 52, 55,$   
 $18$   
 $72+18 \cdot 9=234$ .  
 $6$   
 $(3, 4, 5, 4, 5, 4),$   $(5, 4, 5, 4, 5, 4)$   
 $3$

30. 21 1

21, . 7  
 3 ,  
 ) . :  
 ) , 18  
 6 3 ,  
 . )  
 $\frac{21(21+1)}{2} = 231, \dots$   
 , -  
 ) , 1, 2 12.  
 6 :  
 (3,16,19), (4,9,13), (5,15,20), (6,11,17), (7,14,21) (8,10,18).

31.  $A, B, C, D, E, F, G, H, I, J$

$2 \times 5$

$A$	$B$	$C$	$D$	$E$
$F$	$G$	$H$	$I$	$J$

$G$	$A$	$E$	$F$	$B$
$I$	$C$	$J$	$D$	$H$

.  $A, B, E, C, G$  5 -  
 $D, I, F$  3  
 $H, J$  2 . , -  
 $NZS(2,3,5) = 2 \cdot 3 \cdot 5 = 30$  .

32. 2008 6, 7, 8, 9, ..., 2012, 2013.

?  
 $1+9+9+9=28$ .  
 $1+9=2+8=10$ .  
 , 9

9 9  
 9 6. , 6 9. ,  
 9, -

33.

), ?  
 ) ,  
 ) ?  
 ) ?  
 9  
 9. ,  
 9. ,  
 9 9. , 9

	7	6	5	4	3	2	1
	9	18	27	36	45	54	63
	9	9	9	9	9	9	9
	72	81	99	108	117	126	135
	9	9	18	9	9	9	9

90 -  
 27,  
 91 90 + 27 = 117 9 : 99, 108 117

99 - 18 = 81, 108 - 9 = 99 117 - 9 = 108.  
 ) 135 ,  
 ) 27 ,  
 ) 18 ,  
 )

**3.**

1.  $25$ ,  $42$ ,  $50$ ,  $78$ ,  $105$

$?$   
 $49$   
 $25 + 42 + 3 \cdot 49 = 214$   
 $50$   
 $50$   
 $215$   
 $50$   
 $215$   
 $50$

2.  $11$ ,  $10$ ,  $8$ ,  $?$

$s$ ,  $z$ ,  $c$   
 $s + z = 7$   
 $s + c = 9$   
 $11$   
 $10$   
 $s + z = 7$ ,  $s + c = 9$ ,  
 $z + c = 10$ .  
 $s + z + c = 13$ ,  $s = 3, z = 4, c = 6$ .

3.  $8$

5

?

1, 2, 3, 4,

5, 6 7 . , ,

4 , 7 · 4 = 28 . ,

1 7 , , 29 ,

29 = 7 · 4 + 1,

5

30 .

4. 26

100

. 50

: 1, 3, 5, 7, ..., 99

100.

100

25

: {1,99}, {3,97}, ..., {47,53}, {49,51} .

26 , 25 -

100.

5. 30<sup>100</sup>

.

30<sup>100</sup> 2, 3 5,

30<sup>100</sup> 2<sup>a</sup>3<sup>b</sup>5<sup>c</sup> , a, b, c -

.

(a, b, c) .

(a', b', c') , 0 1,

x' =  $\begin{cases} 0, & a \\ 1, & a \end{cases}$  ,

0 1 : (0,0,0), (1,0,0), (0,1,0), (0,0,1),

(1,1,0), (1,0,1), (0,1,1) (1,1,1), 8 .

,

2 , 3

5 ,

2, 3 5

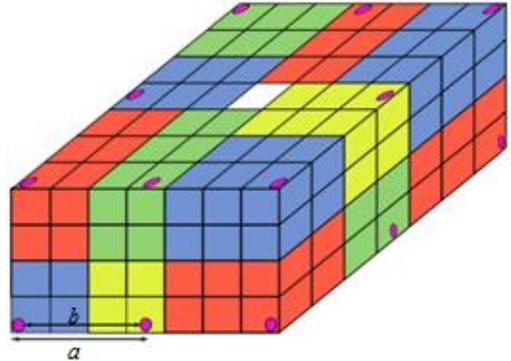
6.

7 m.  
 10 cm,  
 m, 3 m 2 m

( ) -  
 4 m  
 1 m 4 m.  
 16  
 ?  
 17 ?  
 2 m, 3 m 2 m  
 2

$7 \cdot 7 \cdot 4 - 4 = 192 m^3$ .

$192 : 12 = 16$   
 2 m, 3 m 2 m,



$a = 3,5 m,$   
 $b = 3,5 m - 10 cm - 5 cm = 3,35 m .$

2 m, 3 m 2 m  
 ) 17 , 17 . , 16

7.

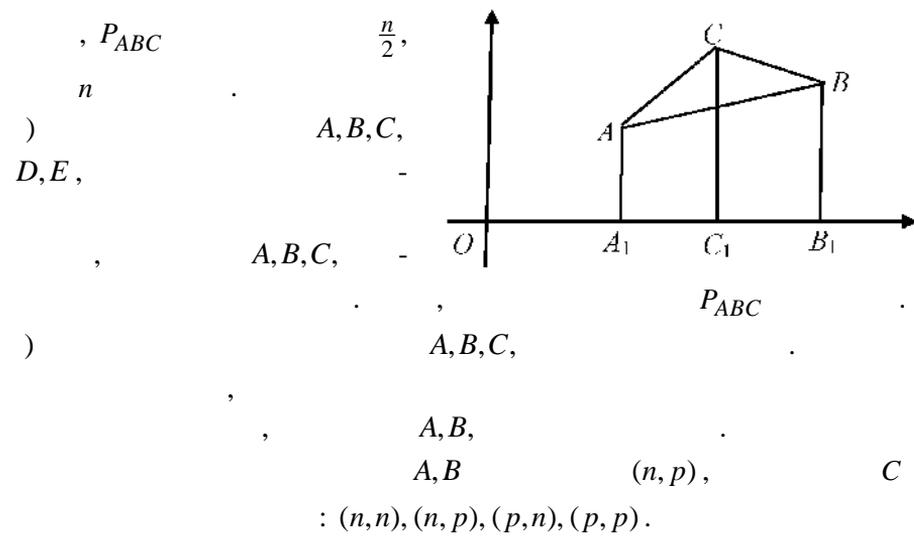
A, B, C, D, E  
 M

$A, B, C, D, E$  :  $M$   
 $\frac{n}{2}, n$  ;  $M$   
 $M$   
 $M$   
 $X(x_1, x_2),$   
 $X_1(x_1, 0) \cdot X(x_1, x_2) \cdot Y(y_1, y_2)$   
 $XX_1Y_1Y$  -  
 $XX_1 \quad YY_1 \quad X_1Y_1$  ( ) .

$$P = \frac{\overline{XX_1 + YY_1}}{2} \cdot \overline{X_1Y_1} = \frac{x_2 + y_2}{2} (y_1 - x_1).$$

$P$   $\frac{n}{2}, n$  . -  
 $x_1 \quad y_1 \quad X \quad Y$  -  
 $P$   $x_2 \quad y_2$   
 $P$  .  
 $P$   $\frac{n}{2}, n$  .  
 $A, B \quad C$   
 ( ) .

$$P_{ABC} = P_{A_1C_1CA} + P_{C_1B_1BC} - P_{A_1B_1BA} \tag{1}$$



(1)

$P_{A_1B_1BA}$

$P_{C_1B_1BC}$   $P_{A_1C_1CA}$

$ABD$   $ABE$ .

8.  $x$  (  $1\text{ cm}$ )

$1\text{ mm}$ .

$x$

11

$\{x\}$

$x$

$x = [x] + \{x\}$ ,  $[x]$

$x$ .

$x$   $[0,1]$ .

$x_1, x_2, \dots, x_{11}$

$[0,1]$  10

$1\text{ mm}$ . 11

$\{x_1\}, \{x_2\}, \dots, \{x_{11}\}$ ,

$\{x_i\}$   $\{x_k\}$

$-0,1 \leq \{x_i\} - \{x_k\} \leq 0,1$ ,

$x_i$   $x_k$

9.  $1\text{ cm}$

:

” “

(

$1\text{ cm}$ ).

$f\text{ cm}$ ,

$3\text{ cm}$ .

$1\text{ cm}$   $[n, n+1)$

$[0,1)$ .

$3\text{ cm}$ ,

$1\text{ cm}$ .

$[0,1),$   
 $\Delta_1 \quad \Delta_2 \quad X \in \Delta_1 \quad Y \in \Delta_2$   
 $[0,1). \quad X$   
 $Y$

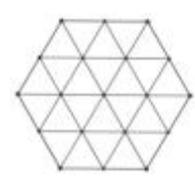
10.  $P = \{1, 2, 3, \dots, 100\}.$   
 $51$   
 $5.$   
 $P \quad 50$

$P_1 = \{1, 6\}, \quad P_2 = \{2, 7\}, \quad P_3 = \{3, 8\}, \quad P_4 = \{4, 9\}, \quad P_5 = \{5, 10\},$   
 $P_6 = \{11, 16\}, \quad P_7 = \{12, 17\}, \quad P_8 = \{13, 18\}, \quad P_9 = \{14, 19\}, \quad P_{10} = \{15, 20\},$   
 $\dots$   
 $P_{46} = \{91, 96\}, \quad P_{47} = \{92, 97\}, \quad P_{48} = \{93, 98\}, \quad P_{49} = \{94, 99\}, \quad P_{50} = \{95, 100\}.$

$50$   
 $P \quad 51$   
 $5,$

11.  $2 \text{ cm}$   $193$   
 $4$   
 $\frac{\sqrt{3}}{4} \text{ cm}^2.$

$193 = 4 \cdot 48 + 1$   
 $49$



$2 \text{ cm},$   
 $1 \text{ cm} ( \quad ).$   
 $24$   $1 \text{ cm}, \dots$

$\frac{\sqrt{3}}{4} \text{ cm}^2.$   $49$   
 $49 = 2 \cdot 24 + 1,$

$$\frac{\sqrt{3}}{4} cm^2.$$

12.  $a, b, c, d$   $a > b > c > d$

$$S = (a-b)(a-c)(a-d)(b-c)(b-d)(c-d).$$

)  $3 | S.$  )  $4 | S.$   
 . )  $3,$  -  
 $0, 1, 2, 3, 4,$  ,  
 $3$  ,  $S$  ,  
 $3 | S.$   
 )  $4,$   $0, 1, 2$   
 $3.$   $4$   
 ,  $4.$  ,  $S$   
 $4 | S.$  ,  
 $4$  ,  $a, b, c, d$  -  
 $4.$  ,  
 , ... ,  $2.$  ,  
 $S$  ,  $2,$   $2 \cdot 2 | s$   
 $4 | S.$

13. ) , 2015  
 4030, 2015  
 ) , 2016  
 4030 ( ) 2015.  
 . ) 2014 1  
 2016.  
 )

$$a_1, a_2, \dots, a_{2016}$$

$$b_1 = a_1,$$

$$b_1 = a_1 + a_2,$$

.....,



4. 0, 0, 0, 1, 1, 1 4.

,  
4.  
4, 4, ,

1) 0, 0, 1, 1. 2,

$0+1+1+2=4,$  3,  $0+0+1+3=4.$

2) 0, 0, 2, 2,  $0+0+2+2=4.$

3) 0, 0, 3, 3. 1,

$0+0+3+1=4,$  2,  $0+3+3+2=8.$

4) 1, 1, 2, 2. 0,

$1+1+2+0=4,$  3,  $1+2+2+3=8.$

5) 1, 1, 3, 3,  $1+1+3+3=8.$

6) 2, 2, 3, 3. 0,

$2+3+3+0=8,$  1,  $2+2+3+1=8.$

16.

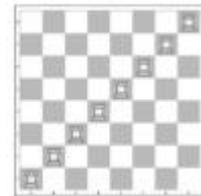
?

8×8,

8

8

9



8

8

8

(

).



$$25 + 75 + 8 = 108$$

3.  $1 \quad 2013.$   
 $3,$   
 $11. \quad ?$   
 $2013:3 = 671 \quad 3$   
 $2013:11 = 183 \quad 11. \quad \text{NZD}(3,11) = 33,$   
 $3 \quad 11 \quad 2013:33 = 61. \quad ,$   
 $671 + 183 - 61 = 793 \quad ,$   
 $2013 - 793 = 1220 \quad .$

4.  $22, \quad 11$   
 $2. \quad ?$   
 $\overline{abcd}.$   $a + b + c + d = 22$   
 $11 \quad a + c - (b + d) \quad 11.$   
 $a + c - (b + d) = \pm 11 \quad , \quad a + c - (b + d)$   
 $a + b + c + d,$   
 $a + c - (b + d) = 0, \dots a + c = b + d = 11.$   
 $a = 2 \quad c = 9 \quad b + d = 11 \quad 8$   
 $2 + 9 = 3 + 8 = 4 + 7 = 5 + 6 = 6 + 5 = 7 + 4 = 8 + 3 = 9 + 2. \quad -$   
 $b = 2, c = 2, d = 2 \quad 8 \quad . \quad 32$

$2,$   
 $a + c = b + d = 11$   
 $2.$   
 $a = b = 2, a = d = 2, b = c = 2 \quad c = d = 2. \quad , \quad 32 - 4 = 28$

5.  $11-$   $9,$   
 $\overline{*2013*2013*}, \quad ($   
 $)$   
 $0, 1, 2 \quad 3?$   
 $12,$   
 $105, \quad 6 \quad 9.$   
 $996,$

$$\frac{996-105}{9} + 1 = 100.$$

$$0, 1, 2, 3, \dots, 6, \dots$$

$$2+1+6=9, \quad 6=3+3+0=2+2+2=1+2+3,$$

6.

$$n = 100a + b, \quad 11 \mid 99a + a + b = 100a + b = n$$

$$99a + a + b = 100a + b = n$$

$$(99990 - 9999) : 11 = 8181$$

7.

$$n^n = (2k+1)^{2k+1} = (2k+1)^{2k} (2k+1)$$

$$n = 2k + 1$$

$$: 1, 9, 25, 49, 81, \dots$$

8.

$$16, 32, 64, 128, 256, 512, \dots$$

$$2, 4, 6, 8, \dots$$

$$27, 81, 243, 729, \dots$$

2	3	
1		
2, 32, 512	27	29, 59, 539
4, 64		
16, 256	3, 243	19, 259, 499
8, 128	1, 81	9, 89, 129, 209
	9, 729	

9. 25

3 2 .

. 25 -

00, 25, 50 75.

00, :

aaa00, aa000, a0a00, a0000,  $a$  0.

3,  $a \in \{1, 2, \dots, 9\}$ , 9

,  $a \in \{3, 6, 9\}$ ,

, 3 .

, 18 00.

, , 25,

3, 5, 2.

, , 25.

, , 50,

, , 5, 0.

55050 50550. 3, -

, , 75,

, , 5, 7.

3

5. 55575 77775, . .

, 22 25 3

10. -

.

.

.

$\overline{abcd}, a \neq 0.$

$d=0, \quad 9 \cdot 19 = 171 \quad ( \quad a \quad 9$   
 $b \quad c \quad 19 \quad 0).$

$d=1, \quad , \dots \quad 1111.$

$d=2, \quad 3 \quad , \quad a,b,c$

1, 1, 2

$d=3, \quad 3 \quad , \quad a,b,c \quad 1,$

1, 3

$d=4, \quad 3+3=6 \quad , \quad a,b,c$

1, 1, 4    1, 2, 2

$d=5, \quad 3 \quad , \quad a,b,c \quad 1,$

1, 5

$d=6, \quad 3+6=9 \quad , \quad a,b,c$

1, 1, 6    1, 2, 3

$d=7, \quad 3 \quad , \quad a,b,c$

1, 1, 7

$d=8, \quad 3+6+1=10 \quad , \quad a,b,c$

1, 1, 8    1, 2, 4    2, 2, 2

$d=9, \quad 3+3=6 \quad , \quad a,b,c$

1, 1, 9    1, 3, 3

$171+1+3+3+6+3+9+3+10+6=215$

11. -

1, 2 3, ,

X, ,

X ,

121, 112, 211, 222

333

1.  $\overline{abc} \quad \overline{pqr}$

0 ( 1) 4 (

1) 1,

1, 2

3 1 9.  $9 > 2 \cdot 4,$

1

$\overline{abc} \quad \overline{pqr},$  .  
5 .

12. 7, 8 9.  
7, 8 9  
3  
3  
3  
 $3 \cdot 3 \cdot 3 = 27$

13. :  
) 1, 2, 3, 4, ) 0, 7, 8, 9, ) 1, 2, 3, 4, 5, 6?  
.  
)  
a 4 . , b, c  
4 . ,  $4 \cdot 4 \cdot 4 = 64$   
1, 2, 3, 4.  
) a 0,  
( 7, 8, 9). b c  
( 0). ,  $3 \cdot 4 \cdot 4 = 48$   
0, 7, 8, 9.  
)  $6 \cdot 6 \cdot 6 = 216$  -  
1, 2, 3, 4, 5, 6.

14. 7, 8, 9 0.  
3 ( 0  
) , 4  
( ) ,  
4 4 . ,  
 $3 \cdot 4 \cdot 4 \cdot 4 = 192$   
7, 8, 9 0.

15.

$2, 4, 6, 8,$ 
 $0, 2, 4, 6$   
 $8,$ 
-  
-  
 $4 \cdot 5 \cdot 5 \cdot 5 = 500.$

16.

$1, 2, 3,$   
 $1.$ 
 $16-$   
-  
 $\frac{a^2 b^2 c^2}{2a^2 b^2 c^2},$   
 $(1 \quad 3).$ 
 $2.$   
-  
 $a, b, c$   
 $2 \cdot 2 \cdot 2 = 8,$ 
-  
 $2 \cdot 8 = 16.$   
-  
 $2 \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 64$ 
-  
 $2 \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 512$ 
 $16-$ 
-

17.

$0,$ 
-  
-  
 $9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 9 \cdot 10^4.$ 
-  
 $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5.$ 
-  
 $9 \cdot 10^4 - 5^5.$

18.

$9 \cdot 10 \cdot 10 = 900$ 
 $p$

,  $n$  ,  $p \in \{0, 2, 4, 6, 8\}$   $n \in \{1, 3, 5, 7, 9\}$  .  
 0,

$$4 \cdot 5 \cdot 5 = 100 .$$

$$5 \cdot 5 \cdot 5 = 125 .$$

$$900 - 100 - 125 = 675 .$$

..  $p$  ,  $n$   
 $p \in \{0, 2, 4, 6, 8\}$   $n \in \{1, 3, 5, 7, 9\}$  .

$\overline{pnn}, \overline{npn}, \overline{nnp}, \overline{npp}, \overline{pnp}, \overline{ppn}$  .

$$\overline{pnn} \quad 4 \cdot 5 \cdot 5 = 100 \quad ($$

0).

$$\overline{npn} \quad 5 \cdot 5 \cdot 5 = 125 .$$

$$\overline{nnp} \quad 5 \cdot 5 \cdot 5 = 125 .$$

$$\overline{ppn} \quad 4 \cdot 5 \cdot 5 = 100 \quad ($$

0).

$$\overline{pnp} \quad 4 \cdot 5 \cdot 5 = 100 \quad ($$

0).

$$\overline{npp} \quad 5 \cdot 5 \cdot 5 = 125 .$$

$$100 + 125 + 125 + 100 + 100 + 125 = 675 .$$

19. 3720147410273 : 2014.

1)

2)

2) ,  
 ( ) 7 .  
 , 1),

2014 4102. 2014 (

4102) , 2014

( 4102)

0. 2014 ( 4102) -

$$2(10^3 + 9 \cdot 10^2 + 9 \cdot 10^2 + 9 \cdot 10^2) = 7400$$

2014 4102 7

7400 2014102014102 4102014102014

2014 4102,

$$7400 - 2 = 7398.$$

20. 1, 3, 5, 7 9, ?

$\overline{ab}$ ,  $a$

1, 3, 5, 7 9,

$a$ ,  $b$

$5 \cdot 4 = 20$

21. 0, 1, 2, 3, ?

1, 2, 3, 0, 1, 2, ?

3 ?

0, 1, 2, 3,

1, 2 3), (

0). ,  $3 \cdot 3 = 9$ .

3

33,  $9 + 1 = 10$ .

22.  $(a+b)(c+d)(e+f) = 315$  -

1 6 .

?

$315 = 3 \cdot 3 \cdot 5 \cdot 7$  ,

$1 + 2 = 3$  -

$5 + 6 = 11$  . -

5, 7 9,  $315 = 5 \cdot 7 \cdot 9$  .

$3 \cdot 2 \cdot 1 = 6$  ,  $5 \cdot 7 \cdot 9$

:



	$(a, e)$ .		$b$	$10$	$d$ ,
-	$c=0$	9	$b$	10	$d$ ,
-	$c=1$	8	$b$	9	$d$ ,
-	$c=2$	7	$b$	8	$d$ ,
-	$c=3$	6	$b$	7	$d$ ,
-	$c=4$	5	$b$	6	$d$ ,
-	$c=5$	4	$b$	5	$d$ ,
-	$c=6$	3	$b$	4	$d$ ,
-	$c=7$	2	$b$	3	$d$ ,
-	$c=8$	1	$b$	2	$d$ .

,

$$10 \cdot 9 + 9 \cdot 8 + 8 \cdot 7 + 7 \cdot 6 + 6 \cdot 5 + 5 \cdot 4 + 4 \cdot 3 + 3 \cdot 2 + 2 \cdot 1 = 330.$$

26. 1, 3

5

?

1, 3 5

( )

( )

).

3 · 2 · 1 = 6.

27. -

?

1, 3, 5, 7 9,

5

5 · 5 · 5 = 125

124

28.

1, 2, 3, 4,

?

---

1, 2, 3, 4, ...

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

29.

?

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151200$$

30.

0, 2, 4, 5 6?

0,

4  
4

2

3 -

1

$$4 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 96$$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

02456,

04526, 06542

120

2, 4, 5 6,

0.

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$120 - 24 = 96$$

31.

) 10,

) 8?  
 . ) 0. ,  
 , 9 ,  
 8 ,  
 7 6 .  
 ,  $9 \cdot 8 \cdot 7 \cdot 6 = 3024$  .

) 7 : 1  
 7, 2 6, 3 5, 5 3, 6 2, 7 1, 8 0.  
 , ,  
 8, 7 6 . ,  
 $7 \cdot 8 \cdot 7 \cdot 6 = 2352$  .

32. , 1, 2, 3, 4, 5 :

) ,  
 ) ?  
 . 2 4 .  
 ) 2 4 , 2 4

(1, 3 5).  
 $4 \cdot 3 \cdot 2 \cdot 1 = 24$  . , 2 4  
 , 2  
 ,  $2 \cdot 24 = 48$  .

)  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$  .  
 ,  
 2 4 ,  
 2 4 . )  
 $120 - 48 = 72$  .

33. 5,  
 1, 2, 3, 4, 5 6 (

), 1 2 ?  
 . 5.  
 -  
 $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$  .  
 4 4 1 2 ,

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$120 - 2 \cdot 24 = 72$$

34.

8, 9 0, 6, 7,  
?

4

( 0), 4 ( 0). , 4 · 4 = 16

4 · 4 · 3 = 48

4 · 4 · 3 · 2 = 96

4 · 4 · 3 · 2 · 1 = 96

4 + 16 + 48 + 96 + 96 = 260 .

35.

0, 1, 2 3.

0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33, 100, ...

2013?

3 · 4 = 12

( 3 4 ) .

3 · 4 · 4 = 48

2000 1 · 4 · 4 · 4 = 64 .

2000, 2001, 2002, 2003, 2010, 2011, 2012 2013,

2013 4 + 12 + 48 + 64 + 8 = 136 - .

36.

7 -

2014?

2014  
2

7 (2005), -

$$\overline{abc} \quad a+b+c=6.$$

$$a+b+c=6$$

$$a+1=x, b+1=y, c+1=z.$$

$$x+y+z=a+b+c+3=9.$$

9

y,

x,

z

$$x+y+z=9.$$

9

$$x+y+z=9$$

8

2

$$\frac{8 \cdot 7}{2} = 28$$

$$28+2=30.$$

37.

{0,1,2,3,4,5}

5?

5

0

5.

0,

5

3

4

$$5 \cdot 4 \cdot 3 = 60$$

5,

4

(0

),

4

3

$$4 \cdot 4 \cdot 3 = 48$$

$$60+48=108.$$

38.

5

0 5.  $\overline{5}$   
 9 ( 0. 1, 2, 3, 4, 5, 6, 7, 8, 9).  
 , , 8  
 , 9 · 8 · 7 = 504 .  
 5.  
 8 ( 1, 2, 3, 4, 6, 7, 8, 9).  
 8 ( 0)  
 , 7 .  
 , 8 · 8 · 7 = 448 .  
 , 504 + 448 = 952 .

39.  $\overline{777}$ ,  
 0, 1, 2, 3 4.  
 . 5, 6, 7, 8 9.  
 ,  $\overline{777}$  7, 8  
 9.  
 8 9,  $2 \cdot 5 \cdot 5 = 50$  ( 5  
 2 , 5  
 ).  
 7, 8 9,  $1 \cdot 2 \cdot 5 = 10$   
 ( 1 , 2  
 5 ).  
 77, 8 9,  
 ... .  
 ,  $50 + 10 + 2 = 62$  .

40.  $\overline{555}$   $\overline{9999}$   
 0, 1, 3, 5, 7,  
 .  
 .  $\overline{570, 571, 573}$   $\overline{7ab}$  .  
 $\overline{7ab}$  a 4 ,  
 b 3 . ,  $3 + 4 \cdot 3 = 15$

0, 1, 3, 5 7,  
 $4 \cdot 4 \cdot 3 \cdot 2 = 96$ .  
 $15 + 96 = 111$ .

41. 30000,  
 2, 3, 4, 5, 3, 4, 5  
 2 ?  
 $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$   
 2, 2, 3, 4, 5, 2,  
 3, 4, 5, 2  $120 : 2 = 60$ .  
 2, 3, 4, 5  
 2, 3, 4, 5, 2  
 $4 \cdot 3 \cdot 2 \cdot 1 = 24$ .  
 30000, 30000,  
 $60 - 24 = 36$ .

42. 816239745 :  
 ) 1 9  
 ) 6, 7, 8, 9 12345,  
 ) 7, 8, 9 123456.  
 ), ) ).  
 ). 6, 7, 8, 9 )  
 8 , 6 9 , 7  
 6 , 8 7 , 9  
 $9 \cdot 8 \cdot 7 \cdot 6 = 3024$   
 ) 1, 2, 3, 4, 5  
 ), . . 7, 8, 9  
 123456.  
 7, 8, 9  $9 \cdot 8 \cdot 7 = 504$ ,

1, 2, 3, 4, 5, 6.

$$3024 - 504 = 2520.$$

43.

$a_1 \neq 0$ .  
 $a_1 = 4$ ,  
 $\frac{6 \cdot 5}{2} = 15$ .  
 $9 \cdot 8 \cdot 7 \cdot 6 = 3024$   
 $15 \cdot 3024 = 45360$   
 $a_1 \neq 4$ ,  
 $\frac{6 \cdot 5 \cdot 4}{6} \cdot 8 \cdot 8 \cdot 7 \cdot 6 = 53760$   
 $45360 + 53760 = 99120$ .

44.

$$\begin{aligned}
 & \overline{abcd} = 4 \cdot \overline{abc} + \overline{bcd} \\
 & \overline{abcd} = 4 \cdot \overline{abc} + \overline{bcd} - a \\
 & \overline{abcd} = 4 \cdot \overline{abc} + \overline{bcd} - a, \quad a, b \neq 0. \\
 & \overline{abcd} = 4 \cdot \overline{abc} + \overline{bcd} - a \\
 & 4, \quad : 12, 16, 20, 24, 28, \\
 & 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96. \\
 & \quad : 12, 24, 36, 48, 60, 72, 84, 96 \\
 & \quad 3, \quad d \\
 & \quad d \in \{0, 3, 6, 9\}. \quad a \\
 & \quad 9, \quad 9 \cdot 8 \cdot 4 = 288 \quad \overline{abcd} \\
 & \quad 3, \quad d \\
 & \quad d \in \{1, 4, 7\} \quad d \in \{2, 5, 8\}. \quad a \\
 & \quad 9, \quad 9 \cdot 14 \cdot 3 = 378 \quad \overline{abcd} \\
 & \quad 288 + 378 = 666 \quad \overline{abcd}
 \end{aligned}$$

45.

$$\begin{aligned}
 & ) 330, \quad ) 4500. \\
 & \cdot ) \quad 330 = 2 \cdot 3 \cdot 5 \cdot 11. \quad 2, 3, 5 \\
 & 11 \\
 & \quad 330 \\
 & \quad 2, 3, 5 \quad 11 \\
 & \quad 330 \quad 2 \cdot 2 \cdot 2 \cdot 2 = 16. \\
 & ) \quad 45000 = 2^3 \cdot 3^2 \cdot 5^4. \quad 360 \\
 & \quad 2^k \quad 2^0, 2^1, 2^2, 2^3, \dots \quad 4, \quad 3^k \\
 & 3^0, 3^1, 3^2, \dots \quad 3, \quad 5^k \quad 5^0, 5^1, 5^2, 5^3, 5^4, \dots \\
 & \quad 5. \quad 45000 \quad 4 \cdot 3 \cdot 5 = 60
 \end{aligned}$$

46.

$$N = 2^8 \cdot 3^8 \cdot 5^8 \cdot 7^8 \cdot 11^8 \cdot 13^8 \cdot 17^8 \cdot 19^8$$

$$8 = 7 + 1 = (1+1) \cdot (3+1) = (1+1) \cdot (1+1) \cdot (1+1),$$

- $n$  :  
 1)  $n = p^7$ ,  $p$  8 ,  
 2)  $n = pq^3$ ,  $p, q$   $8 \cdot 7 = 56$  ,  
 3)  $n = pqr$ ,  $p, q, r$   $(8 \cdot 7 \cdot 6) : 6 = 56$  -  
 ,  $8 + 56 + 56 = 120$   $N$  8 -

47. :

- 30,  
 - 3  
 - 300 .

$$300 = 2^i 3^j 5^k$$

$$(i+1)(j+1)(k+1) = 300,$$

$i, j, k \geq 1$ .

$$300 = 2^2 \cdot 3 \cdot 5^2$$

1.  
 $300 = 2^2 \cdot 3 \cdot 5^2$   
 $3 \cdot 3 \cdot 6$   
 $(3, 3, 6)$   
 $(4, 1, 1)$  3  
 $(2, 2, 1)$ . ,  
 $300$   
 $3 \cdot 6 \cdot 6 = 108$

1. 1, 18  
 $300$   
 $(300)$ , . . .  
 $3 \cdot 18 = 54$ . , 54  
 , 3,  
 $54 - 3 = 51$ .  
 , 300

$$108 - 51 = 57,$$

48.  $(a, b, c)$  -

$$abc = 9000.$$

$$9000 = 2^3 \cdot 3^2 \cdot 5^3.$$

2  $a, b, c$  :  
 $(3, 0, 0), (0, 3, 0), (0, 0, 3), (2, 1, 0), (2, 0, 1), (0, 2, 1), (1, 2, 0), (0, 1, 2), (1, 0, 2), (1, 1, 1),$   
 10 , 10

$$\frac{5}{2} a, b, c$$

:

$(2, 0, 0), (0, 2, 0), (0, 0, 2), (1, 1, 0), (1, 0, 1), (0, 1, 1).$

6

$$10 \cdot 10 \cdot 6 = 600.$$

49.  $(a, b, c, d)$

$$abcd = 15000.$$

$$15000 = 2^3 \cdot 3 \cdot 5^4.$$

„2“ „ ( ) „  
 $a, b, c, d$ .

„ “ „ “

$$\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$$

„5“ „ ( )  
 $a, b, c, d$ .

„ “ „ “

$$\frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35. \quad 3$$

4

++++ ( ), ++-- ( ) ---- ( )  
 ), 8

$$20 \cdot 35 \cdot 4 \cdot 8 = 22400.$$

50.

$(a,b,c)$   $a^4 b^2 c = 54000?$   
 $54000 = 2^4 3^3 5^3,$   
 $a = 2^{a_1} 3^{a_2} 5^{a_3}, b = 2^{b_1} 3^{b_2} 5^{b_3} \quad c = 2^{c_1} 3^{c_2} 5^{c_3}.$   
 $4a_1 + 2b_1 + c_1 = 4$   $4a_2 + 2b_2 + c_2 = 3$   $4a_3 + 2b_3 + c_3 = 3,$   
 $(1,0,0), (0,2,0), (0,1,2)$   $(0,0,4),$   $(0,1,1), (0,0,3).$   
 $2$   $4 \cdot 2 \cdot 2 = 16$

51.

$2$   $3.$   $1,$   
 $1 \quad 3$   $.$   
 $?$   
 $2.$   $8$   $3$   $,$   $1$   
 $2.$   $2$   $8$   $.$   $\frac{8 \cdot 7}{2} = 28$   $2$   
 $,$   $3$   $,$   $3$   
 $,$   $\dots$   $28.$   
 $3$   $,$   
 $5$   $,$   
 $7$   $.$   $-$   
 $2$   $2$   $-$   
 $7$   $,$   $\frac{7 \cdot 6}{2} = 21$   $.$   $,$   $3$   
 $,$   $,$   $\dots$   $21$   
 $.$   
 $2 \cdot 28 + 8 \cdot 21 = 224$

52.

$\frac{p}{q}$   $(0,1)$   $M(p,q) = pq$   
 $M(p,q) = 25!.$   
 $25!$   $9$   $: 2,$   
 $3, 5, 7, 11, 13, 17, 19$   $23.$

$$\frac{p}{q}$$

$$\frac{p}{q}$$

$$M(p, q) = 25!$$

$$2^9$$

(0,1)

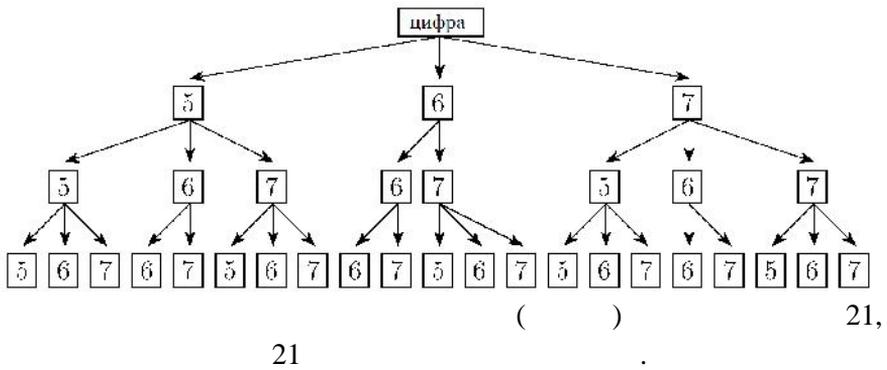
(0,1)

$$\frac{2^9}{2} = 2^8 = 256$$

53.

5, 6 7,  
6.

5



54.

3?

$$A = \{1, 4, 7\}, \quad B = \{2, 5, 8\}$$

$$C = \{0, 3, 6, 9\}$$

3

AAA

$$3 \cdot 2 \cdot 1 = 6,$$

BBB

$$3 \cdot 2 \cdot 1 = 6,$$

CCC

$$3 \cdot 3 \cdot 2 = 18,$$

ABC, ACB, BAC,

BCA

$$3 \cdot 3 \cdot 4 \cdot 4 = 144$$

CAB, CBA

$$3 \cdot 3 \cdot 3 \cdot 1 = 54.$$

3

$$6 + 6 + 18 + 144 + 54 = 228,$$

$$9 \cdot 9 \cdot 8 = 648.$$

$$648 - 228 = 420, \\ 3, \quad 421 \\ 3.$$

55.

$$3. \quad : \\ A = \{0, 3, 6, 9\}, B = \{1, 4, 7\}, C = \{2, 5, 8\}. \\ 3.$$

$$3. \quad 3$$

:

- 1)  $B \subset C, \quad A,$
- 2)  $B \subset C, \quad A,$
- 3)  $B, \quad C,$

A.

0

0,

$$1) \quad 3 \cdot 3 \cdot 3 = 27, \quad 2) \quad - \\ 3 \cdot 3 \cdot 1 = 9 \quad 3) \quad 3 \cdot 2 = 6 \quad - \\ 27 + 9 + 6 = 42 \quad - \\ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$42 \cdot 120$$

0,

$$1) \quad 3 \cdot 3 \cdot 1 = 9, \\ 2) \quad 3 \cdot 3 \cdot 3 = 27 \quad 3) \\ 3 \cdot 2 = 6, \quad 9 + 27 + 6 = 42, \\ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 - 4 \cdot 3 \cdot 2 \cdot 1 = 96, \quad 42 \cdot 96$$

$$42 \cdot 120 + 42 \cdot 96 = 9072$$

3

56.

$$n \geq 3$$

A

$$n -$$

3,

10.

B

n -

4.

$A = 3 \cdot 9 \cdot 10^{n-2} = 270 \cdot 10^{n-3}$   
 $B = 25 \cdot 9 \cdot 10^{n-3} = 225 \cdot 10^{n-3}$   
 $B = 225 \cdot 10^{n-3} < 270 \cdot 10^{n-3} = A$

57.  $n \geq 3$ .  $A = 3 \cdot 10^{n-2}$ ,  $B = 5 \cdot 7 \cdot 10^{n-2}$ .  
 $A = 27 \cdot 10^{n-2}$ ,  $B = 2 \cdot 9 \cdot 10^{n-2} = 18 \cdot 10^{n-2}$ .  
 $B > A$ ,  $9 \cdot 10^{n-2}$ .  
 $t = \overline{b_1 b_2 \dots b_{n-1} 0}$ .  
 $t = 0, 1, 2, 3, 4, 5, 6$   
 $t = 7, 6, 5, 4, 3, 2, 9, 1, 8$ .  
 $t = 0, 1, 2, 3, 4, 5, 6, 7$   
 $t = 0, 10, 20, 30, 40, 50, 60, 0, 3, 6, 2$

5, 1, 4. , 7 (n-1)-  
n-

7, 5. n-  
7, 5 9 · 10<sup>n-2</sup>.

58.

1, 2, 3, ..., 10  
 $a_1, a_2, \dots, a_{10}$   $a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots,$   
 $a_1 + a_2 + \dots + a_{10}$  3.  
 . 1, 4, 7 10 1 3, -  
 2, 5 8 2 3 3, 6 9  
 3. 3, 6 9  
 3, 3.  
 2 3,  
 2, 1, 2, 1, 2 .  
 2  
 1, . , 1, 1, 2, 1, 2, 1, 2.

1, 4, 7 10, 2, 5 8,  
 4! · 3! . 3,  
 3 7 , 6 8  
 9 9 . , 4! · 3! · 9 · 8 · 7 .

5.

1.

?

5  $A B$ ,  $AB BA$  6.5

$$\frac{6 \cdot 5}{2} = 15$$

$A_1, A_2, A_3, A_4, A_5, A_6$ .

- $A_1A_2, A_1A_3, A_1A_4, A_1A_5, A_1A_6,$
- $A_2A_3, A_2A_4, A_2A_5, A_2A_6,$
- $A_3A_4, A_3A_5, A_3A_6,$
- $A_4A_5, A_4A_6,$
- $A_5A_6.$

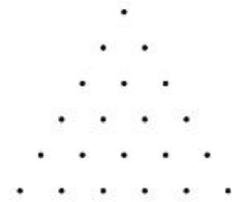
$$5 + 4 + 3 + 2 + 1 = 15.$$

2.

$$\frac{21 \cdot 20}{2} = 210$$

210

21



$$\frac{6 \cdot 5}{2} = 15$$

6 6

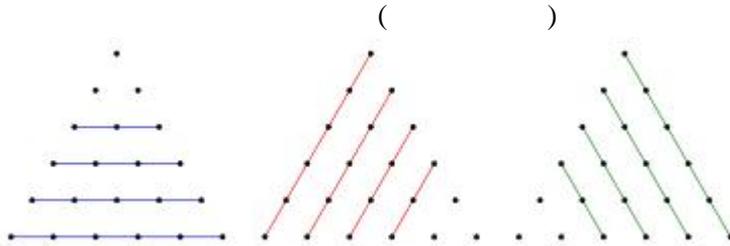
$$15 - 5 = 10$$

$$\frac{5 \cdot 4}{2} - 4 = 6$$

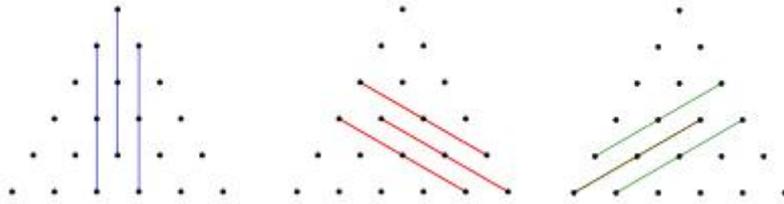
$$\frac{4 \cdot 3}{2} - 3 = 3$$

1

$$10 + 6 + 3 + 1 = 20$$



$$3 \cdot 3 = 9 \text{ ( )}.$$



$$210 - 3 \cdot 20 - 3 \cdot 3 = 210 - 69 = 141.$$

3.

$$\frac{5 \cdot 4}{2} = 10.$$

$A_1, A_2, A_3, A_4, A_5.$

$A_1A_2A_3, A_1A_2A_4, A_1A_2A_5, A_1A_3A_4, A_1A_3A_5, A_1A_4A_5,$   
 $A_2A_3A_4, A_2A_3A_5, A_2A_4A_5,$   
 $A_1A_4A_5.$

$$6 + 3 + 1 = 10.$$

$A_1, A_2, A_3, A_4, A_5$

$(X, Y, Z).$

5

4

3

$$5 \cdot 4 \cdot 3.$$

$$3 \cdot 2 \cdot 1.$$

$$\frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10.$$

4.

6

)  
)

?

. )

A

6

B

5

AB

BA

$$\frac{6 \cdot 5}{2} = 15.$$

)

A, B C

6,

5

4

$(A, B, C), (C, A, B), (B, C, A), (A, C, B), (B, A, C), (C, B, A)$

$$\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$$

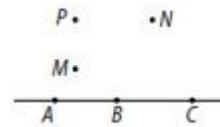
5.

p

p ·

?

$\frac{6 \cdot 5}{2} = 15$ .  
 $A, B, C$   
 $M, N, P$  3  
 $M, N, P$   $A, B, C$   
 9  
 $1 + 3 + 9 = 13$ .



6. 8 , ?

$\frac{8 \cdot 7}{2} = 28$   
 $4 \cdot 2 = 8$   
 3  
 $28 - 8 = 20$ .

7.  $AC$   $BC$   $ABC$  9

?  
 $A$   $10 + 9 + 8 + \dots + 2 + 1 = 55$   
 $BC$ , 9  
 $B$   
 $AC$ ,  $A$   $10 \cdot 55 = 550$ .  
 $B$ .  
 $A$   $B$   
 $10 \cdot 10 = 100$ ,  $2 \cdot 550 - 100 = 1000$

8.  $AC$   $BC$   $ABC$   $n$

?  
 $A$   $\frac{(n+2)(n+1)}{2}$   
 $BC$ ,  $n$   
 $n - B$   $AC$ .

$$A \quad \frac{(n+2)(n+1)^2}{2}.$$

$$B.$$

$$A \quad B$$

$$(n+1)^2.$$

$$2 \cdot \frac{(n+2)(n+1)^2}{2} - (n+1)^2 = (n+1)^3$$

9.  $5$  .

$$20 \quad , \quad 3$$

$$\frac{20 \cdot 19 \cdot 18}{3 \cdot 2} = 1140$$

$$5$$

$$( \quad ) 3$$

$$\frac{5 \cdot 4 \cdot 3}{3 \cdot 2} = 10$$

$$4 \quad ( \quad ),$$

$$4 \cdot 10 = 40,$$

$$1140 - 40 = 1100.$$

10.  $1 \text{ cm}$

$$2 \text{ cm}.$$

$$A$$

$$A$$

$$1, 1, \sqrt{3}, \sqrt{3}, 2, 2, \sqrt{5}, \sqrt{5}, \sqrt{7}, \sqrt{7}, 2\sqrt{2} \text{ cm}.$$

$$\sqrt{3} \text{ cm}.$$

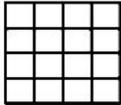
$$( \quad ) \quad 12 \cdot 4 + 4 = 52.$$

$$\frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220,$$

$$220 - 52 = 168.$$

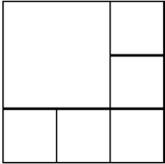
11.

$1\text{ cm}$ ,  $4\text{ cm}$ ,  $16$   
 $16$   
 $16 + 9 + 4 + 1 = 30$   
 $16 \cdot 4 \cdot 1 + 9 \cdot 4 \cdot 2 + 4 \cdot 4 \cdot 3 + 1 \cdot 4 \cdot 4 = 64 + 72 + 48 + 16 = 200\text{ cm}$



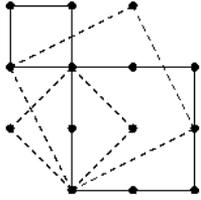
12.

$3 \times 3$ ,  $1 \times 3$ ,  $5$ ,  $1 \times 1, 1$ ,  $1 \times 2$ ,  $2 \times 3$ ,  $2 \times 2, 1$ ,  $15$   
 $\frac{7}{15}$



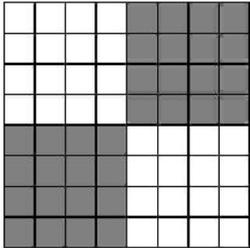
13.

$4$ ,  $14$ ,  $?$ ,  $16$   
 $2$ ,  $7$ ,  $2 \times 2, 4$ ,  $1 \times 1, 2$ ,  $\sqrt{2} \times \sqrt{2}$ ,  $\sqrt{5} \times \sqrt{5}$ ,  $7 + 2 + 4 + 2 = 15$



14.

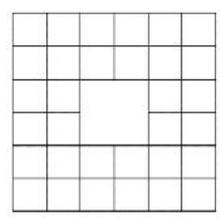
$8 \times 8$ ,  $2 \times 2, 4 \times 4, 6 \times 6, ?$ ,  $13$   
 $2 \times 2$



$4 \times 4$   
 $9$   
 $6 \times 6$   
 $5$   
 $13 + 9 + 5 = 27$

15.

.( .)



1)

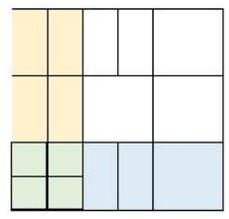
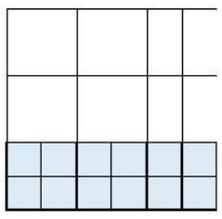
2)

$3$   
 $3 \cdot 3 \cdot 3 \cdot 3 = 81$

$6 \times 2$  ( )  
 : , ,

( ) ,  
 $21$  ( )  
 ).

$3 \cdot 21 = 63$ .



$4 \cdot 63$

$2 \times 2$

( ) .

9

$$4 \cdot 63 - 4 \cdot 9 = 216 .$$

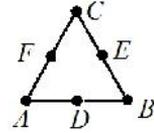
$$81 + 216 = 297 .$$

16.

$D, E, F$

$A, B, C, D, E, F$

$ABC$



$ABC, DEF$

$ABC, DEF$

$ABC$

$AB$

$AC$

$BC, \dots$

2

$BC, CA$

$$3 \cdot 2 = 6$$

$DE$

$DEF$

$A, B, C, \dots$

$EF$

$FD$

$$3 \cdot 3 = 9$$

$$2 + 6 + 9 = 17$$

17.

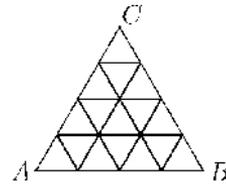
$ABC$

16

)

$87 \text{ cm}^2$

$\triangle ABC$



16

7

4

3

9

1

$\dots \triangle ABC$

16

$$16 \cdot 1 + 7 \cdot 4 + 3 \cdot 9 + 1 \cdot 16 = 87$$

$$87 : 87 = 1 \text{ cm}^2$$

$\triangle ABC$

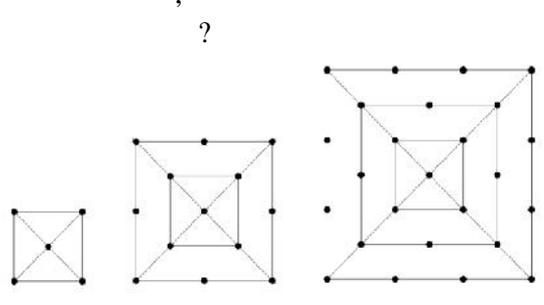
$$16 \cdot 1 = 16 \text{ cm}^2$$

18. , 2009, -  
 , -  
 2012, -

•  $T$   $a, b, c$  2009.  
 $a+1, b+1, c+1$   $T_1$  2012.  
 $S_1$   $x, y, z$  2012.  
 $x, y, z$  1,  $z=1$ ,  
 $1 = z > |x - y| \geq 0$ ,  
 $x, y \in \mathbb{N}$ ,  $x = y$   $2x = 2011$ ,  
 $x-1, y-1, z-1$  .

$(x-1) + (y-1) \leq z-1$ ,  $x+y > z$   $x+y = z+1$ ,  
 $2z+1 = 2012$ ,  $x-1, y-1, z-1$   
 2009.  
 , 2009, -  
 2012 -

19. 5 , 13 , -

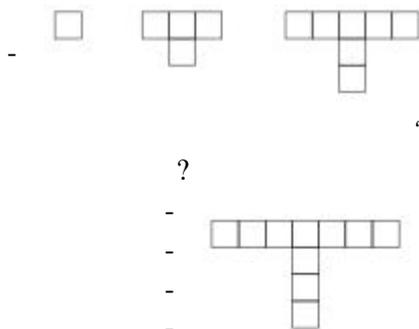


4 8

4

$$\begin{aligned}
 1 + 4 \cdot 1 + 4 \cdot 2 + 4 \cdot 3 + \dots + 4 \cdot 23 &= 1 + 4 \cdot (1 + 2 + 3 + \dots + 23) \\
 &= 1 + 4 \cdot \frac{23 \cdot (23 + 1)}{2} \\
 &= 1105
 \end{aligned}$$

20.



?

$n -$  ?

1, 4, 7 10

$a_1 = 1,$

$d = 3$

$n -$

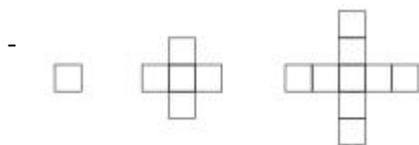
$a_1$

$d$   $n - 1$  ,  $a_n = a_1 + (n - 1)d$  ,

$$a_n = 1 + 3(n - 1) = 3n - 2 .$$

$$a_{1000} = 3 \cdot 1000 - 2 = 2998 .$$

21.



?

?

$n -$

?

13

$$a_1 = 1,$$

$$d = 4$$

$n -$

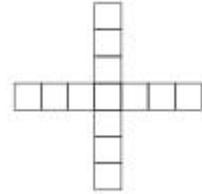
$$a_n = a_1 + (n-1)d,$$

$n - 1$

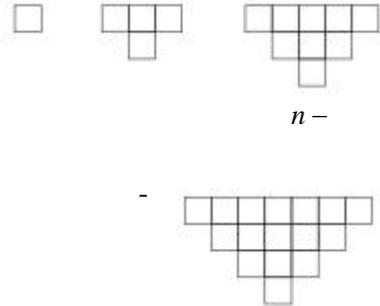
$$a_n = 1 + 4(n-1) = 4n - 3.$$

$$a_{100} = 4 \cdot 100 - 3 = 397.$$

1, 5, 9



22.



$$a_1 = 2 \cdot 1 - 1,$$

$$a_2 = 1 + 3 = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) = 2 \cdot (1 + 2) - 2,$$

$$a_3 = 1 + 3 + 5 = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + (2 \cdot 3 - 1) = 2 \cdot (1 + 2 + 3) - 3,$$

$$a_4 = 1 + 3 + 5 + 7 = (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + (2 \cdot 3 - 1) + (2 \cdot 4 - 1) \\ = 2 \cdot (1 + 2 + 3 + 4) - 4$$

$$\dots, \quad n - \quad n$$

$$i - \quad i - \quad 2i - 1.$$

$n -$

$$a_n = 1 + 3 + 5 + \dots + (2i - 1) + \dots + (2n - 1)$$

$$= (2 \cdot 1 - 1) + (2 \cdot 2 - 1) + (2 \cdot 3 - 1) + \dots + (2i - 1) + \dots + (2n - 1)$$

$$= 2 \cdot (1 + 2 + 3 + \dots + i + \dots + n) - n$$

$$= 2 \cdot \frac{n(n+1)}{2} - n = n(n+1) - n = n^2.$$

23.

$a_1 = 1$ ,  $a_2 = 5$ ,  $a_3 = 13$ ,  $a_4 = 25$ .

$a_n = n^2 + (n-1)^2$ .

$a_1 = 1 = 1^2 + (1-1)^2$

$a_2 = 5 = 2^2 + (2-1)^2$ .

$a_3 = 13 = 3^2 + (3-1)^2$ .

$a_4 = 25 = 4^2 + (4-1)^2$ .

$a_k = k^2 + (k-1)^2$ .

$a_{k+1} = (k+1)^2 + k^2$ .

$a_{k+1} - a_k = (k+1)^2 + k^2 - (k^2 + (k-1)^2) = 2k + 1$ .

$a_{k+1} = a_k + 2k + 1$ .

$a_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_n - a_{n-1})$ .

$a_n = 1 + (2 \cdot 1 + 1) + (2 \cdot 2 + 1) + \dots + (2(n-1) + 1)$ .

$a_n = 1 + 2(1+2+\dots+(n-1)) + (n-1)$ .

$a_n = 1 + 2 \cdot \frac{(n-1)n}{2} + (n-1) = 1 + n(n-1) + (n-1) = n^2 + (n-1)^2$ .

$a_n = n^2 + (n-1)^2$ .

$a_1 = 1 = 1^2 + (1-1)^2$

$a_2 = 5 = 2^2 + (2-1)^2$

$a_3 = 13 = 3^2 + (3-1)^2$

$a_4 = 25 = 4^2 + (4-1)^2$

$a_k = k^2 + (k-1)^2$

$a_{k+1} = (k+1)^2 + k^2$

$a_{k+1} - a_k = (k+1)^2 + k^2 - (k^2 + (k-1)^2) = 2k + 1$

$a_{k+1} = a_k + 2k + 1$

$a_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_n - a_{n-1})$

$a_n = 1 + (2 \cdot 1 + 1) + (2 \cdot 2 + 1) + \dots + (2(n-1) + 1)$

$a_n = 1 + 2(1+2+\dots+(n-1)) + (n-1)$

$a_n = 1 + 2 \cdot \frac{(n-1)n}{2} + (n-1) = 1 + n(n-1) + (n-1) = n^2 + (n-1)^2$

$a_n = n^2 + (n-1)^2$ .

$$\begin{aligned}
 a_{k+1} &= a_k + 2(2k-1) + 2 = k^2 + (k-1)^2 + 4k \\
 &= k^2 + 2k + 1 + k^2 = (k+1)^2 + (k+1-1)^2.
 \end{aligned}$$

$$(1) \quad n = k + 1,$$

$n$ .

24.

?

-

-

?

$n -$

?

.

$a_1 = 1$

$3 = 2 \cdot 1 + 1$

1

.

$a_2 = (2 \cdot 1 + 1)^2 - 4 \cdot 1 = 9$

5 = 2 \cdot 2 + 1

2

.

$a_3 = (2 \cdot 2 + 1)^2 - 4 \cdot 2 = 17$

7 = 2 \cdot 3 + 1

3

.

$a_4 = (2 \cdot 3 + 1)^2 - 4 \cdot 3 = 37$

.

$a_5 = (2 \cdot 4 + 1)^2 - 4 \cdot 4 = 65$ .

,

$n -$

$a_n = (2(n-1) + 1)^2 - 4(n-1) = (2n-1)^2 - 4n + 4$

$= 4n^2 - 4n + 1 - 4n + 4 = 4n^2 - 8n + 5,$

$n = 1.$

25.

$1 \text{ cm}$

$384 \text{ cm}^2$

(

)

)

?

?

)

$5 \text{ cm} ?$

---

•

$$384 = 2^7 \cdot 3.$$

?

6.

1. ( ): , ):

$$\begin{array}{r} ABBCB \\ +BCADA \\ \hline DBDDD \end{array}$$

,  $D \neq 0$   $C + D = D$   $C = 0$ .  
 $A + B = D < 10$ .  
 $D$  3, 4, 5, 6, 7, 8 9.

A B :

D	A+B
3	1+2, 2+1
4	1+3, 3+1
5	1+4, 2+3, 3+2, 4+1
6	1+5, 2+4, 4+2, 5+1
7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1
8	1+7, 2+6, 3+5, 5+3, 6+2, 7+1
9	1+8, 2+7, 3+6, 4+5, 5+4, 6+3, 7+2, 8+1

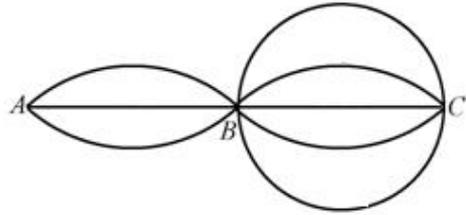
$$2+2+4+4+6+6+8=32$$

2. 1, 2, 3, 4, 5 6 .

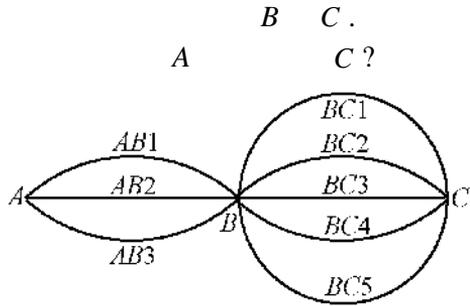
$$-1-2-3-4-5-6=-21 \quad 1+2+3+4+5+6=21.$$

$$-21 \quad 21.$$

3. A, B C -  
 -  
 .  
 A C -  
 ,  
 A C  
 B. -  
 A B,



.  
 A B -  
 AB1, AB2 AB3,  
 B C  
 BC1, BC2, BC3,  
 BC4 BC5. -  
 A C :



AB1 - BC1, AB1 - BC2, AB1 - BC3, AB1 - BC4, AB1 - BC5,  
 AB2 - BC1, AB2 - BC2, AB2 - BC3, AB2 - BC4, AB2 - BC5,  
 AB3 - BC1, AB3 - BC2, AB3 - BC3, AB3 - BC4, AB3 - BC5.

, A C 15 .  
 . A B 3 . B  
 C 5 . A C B  
 3 · 5 = 15 .

4. A B  
 4 , B C  
 3 . A  
 B C.  
 . A B, -  
 B C. , A C,  
 B, 3 · 4 = 12 .

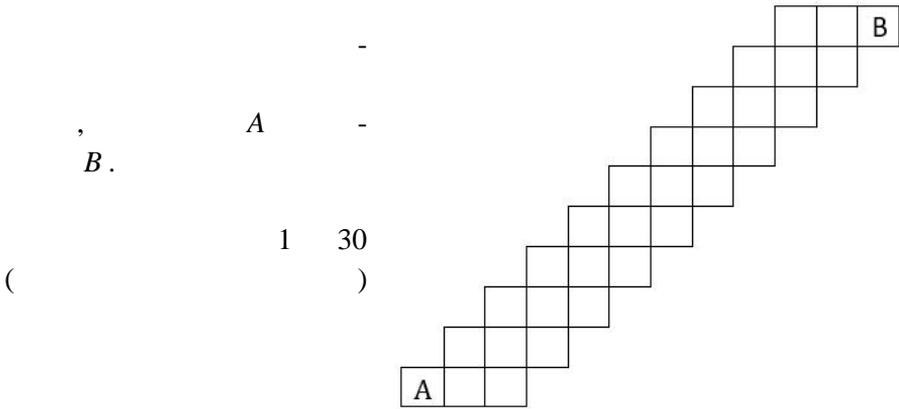
5. 8 .  
 ,  
 , ? -

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256.$

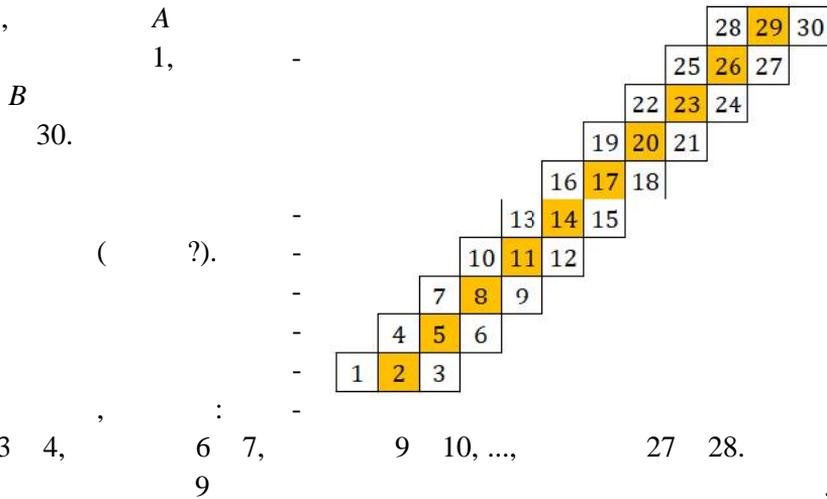
( ),

$256 - 2 = 254.$

6.



?



$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9 = 512.$

7. 10

?

..., 9     10     0.     ,     2     0, 1, 2, 1, : 0

$$\underbrace{2 \cdot 2 \cdot \dots \cdot 2}_{10} = 2^{10}.$$

$$2^{10} - 1 - 10 = 1013.$$

8.     1, 2, 3, 4, 5, 6, 7, 8     9.     -

$$9 \cdot 9 \cdot 9 \cdot 9 = 6561.$$

9.     ?     -

   ?     -

   )     -

   , ...     5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 15625.

$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4096$$

$$6 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 6144.$$

$$15625 - (4096 + 6144) = 5385.$$

10.     ,     2500     .

2023

?

$$4 \cdot 3^{2022}$$

3

11.

2023

?

4

$$4 \cdot 3^{2022}$$

12.

3

5

3

5

3

$$3^5 = 243$$

$$3 \cdot 2^5 = 96$$

$$3 \cdot 1^5$$

$$243 - 96 + 3 = 150$$

13.

?

5

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$$

$$120 + 120 = 240.$$

14.

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$2 \cdot 24 = 48.$$

15.

$$3 \cdot 2 \cdot 1 = 6$$

16.

$$2 \cdot 2 = 4$$

$$4 \cdot 2 \cdot 1 = 8$$

17.

$$3 \cdot 2 \cdot 1 = 6$$

$$2 \cdot 6 = 12$$

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$24 - 12 = 12$$

18.

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

19.

$$25 \cdot 24 \cdot 23 = 13800$$

20.

$$10 \cdot 5 \cdot 5 = 250$$

10. :  
 ,  
 $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$  , , ... 120  
 $120 \cdot 120 = 14400$  .  
 $2 \cdot 14400 = 28800$  .

21. :  
 ) 1 2 ,  
 ) 1 2 ?  
 . ) 1 2 .  
 (1,2)  
 $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$  . 1 2  
 $2 \cdot 8!$   
 )  $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 9!$   
 1 2 , ...  $2 \cdot 8!$   
 $9! - 2 \cdot 8! = 9 \cdot 8! - 2 \cdot 8! = 7 \cdot 8!$  1 2 ,

22. ?  
 ) ,  
 ) ? ,  
 ) ? ,  
 ) ?  
 . )  
 $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$  .  
 ) , ,

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

$$2 \cdot 720 = 1440.$$

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320,$$

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 - 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320 - 5040 = 35280.$$

23.  $12$

$$10!$$

$$10! \cdot 3! = 21772800.$$

24.  $(5 \cdot 4) : 2 = 10$

$$2 \cdot 5 = 10$$

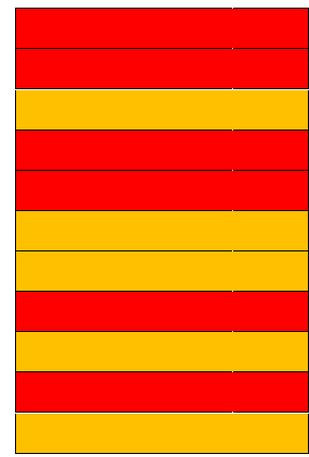
$$10 + 10 = 20$$

25.

?  
 .  
 ),  
 ),  
 ( )  
 ),  
 ( )  
 4 · 3 · 2 · 1 = 24

26. , , 2 , 2 1 3  
 .  
 ?

.  
 4 · 3 · 2 · 1 = 24  
 , 24 : 2 = 12  
 :



( )  
 ( ) ,  
 $6 \cdot 24 + 5 \cdot 12 = 204$

27. 7 5 .  
 , 3 2 ?  
 .

$$7 \cdot 6 \cdot 5 = 210$$

$$3 \cdot 2 \cdot 1 = 6$$

$$210 : 6 = 35$$

$$(5 \cdot 3) : (2 \cdot 1) = 10$$

$$35 \cdot 10 = 350$$

28.  $50, 1, 2$  : 5, 10, 20

$$85 \text{ ?}$$

$$5 + 10 + 20 + 50 = 85$$

$$2$$

$$\frac{43}{2} = 6$$

$$\frac{43}{2} = 6$$

$$1 \cdot 2 \cdot 4$$

$$6 + 6 + 4 = 16$$

29.

$$?$$

$$9, 8, 7, 10$$

$$10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

5040

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

24

$5040 : 24 = 210$

30.

?

(

),

:

:

$4 + 4 = 8$

31.

300

$n$

$\frac{n(n-1)}{2}$

$\frac{n(n-1)}{2} = 300$

$n(n-1) = 600$

$n = 25$

$25 - 2 + 5 = 7$

32.

( )

?

$$( \quad , \quad , \quad ),$$

$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

4,

$$4 \cdot 81 = 324.$$

33.

36,

?

9.

9,

0

9.

, ... = 0

= 9.

4,

4

{2, 4, 6, 8}.

$$4 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 20160$$

34.

6

, 6

6

:

) 6

3

?

) 3

6

?

. )

6

6

$$6 \cdot 6 = 6^2$$

$$5^2$$

$$2^2 \cdot 4^2 \cdot 1^2 \cdot 3^2 = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)^2 = 720^2 = 518400$$

)  $\frac{6 \cdot 5}{2} = 15$

$$15 \cdot 15 \cdot 15 = 15^3$$

$$\frac{4 \cdot 3}{2} = 6$$

$$6 \cdot 6 \cdot 6 = 6^3$$

$$15^3 \cdot 6^3$$

$$3 \cdot 2 \cdot 1 = 6$$

$$\frac{15^3 \cdot 6^3}{6} = 121500$$

35.  $12 \cdot 8 \cdot 2$

1)  $12 \cdot 8 \cdot 2$

$$\frac{8 \cdot 7}{2} = 28$$

$$12 \cdot 28 = 336$$

2)  $\frac{12 \cdot 11}{2} = 66$

$$1 \cdot 66 = 66$$

$$66 \cdot 8 = 528$$

3)

$$\frac{12 \cdot 11}{2} = 66,$$

$$\frac{8 \cdot 7}{2} = 28$$

$$66 \cdot 28 \cdot 2 = 3696$$

$$336 + 528 + 3696 = 4560$$

36.

? ( / )

(xxxxy, xxxyy, xxyxy, xyxyx, xxyyy, xyyyy).

$$3 + 1 + 1,$$

(xxxzy, xxyxz).

(xyyyz, xyxyz, xyxzy, yxxyz).

$$3(1 + 6 + 2 + 4) = 39$$

37.

8!

A, B, C, D.

X.

ABCDXXX.

$$7! : 3! = 840.$$

$$8! : 840 = 33868800.$$

38.

, 6

8

16

$$16 \cdot 15 \cdot 14 \cdot \dots \cdot 3 \cdot 2 \cdot 1 = 20922789888000.$$

2.

6

6

6

6

6

6

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

8

8

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320.$$

16

360360

39.

?

\* \* \* \*

\*

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

10

: 123, 124, 125,

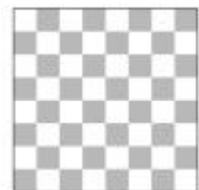
134, 135, 145, 234, 235, 245, 345.

$$24 \cdot 10 = 240$$

40.

8 × 8

?



32

$$32 - 4 - 4 = 24$$

$$32 \cdot 24 = 768$$

41.

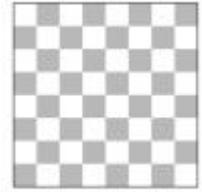
?

$$8 \cdot 8 = 64$$

$8 \times 8$

63

$$64 \cdot 63 = 4032$$



4

27

12-

25

20-

23

28-

21

$$28 \cdot 21 + 20 \cdot 23 + 12 \cdot 25 + 4 \cdot 27 = 1456$$

42.

○

\*

		○		
		*		

\*?

○

○

1,

0.

0	0	1	0	0
0	1	1	1	0
1	2	3	2	1
3	6	7	6	3
9	16	19	16	9
25	44	51	44	25

\*

○

51

43.

8

1, 2, 3, 4, 5, 6, 7, 8.

1 8

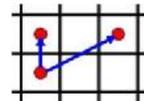
9 9

1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8
		2	5	9	14	20	27	35
			5	14	28	48	75	110
				14	42	90	165	275
					42	132	297	572
						132	429	1001
							429	1430
								1430

1430,

44.

9×9.



?

1		8	28		56	70
1		7	21		35	35
1		6	15		20	15
1		5	10		10	5
1		4	6		4	1
1		3	3		1	
1		2	1			
1		1				
●						

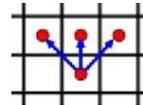
1.

$$1 + 8 + 28 + 56 + 70 = 163$$

45.

$7 \times 7$

( )



?

44	89	126	141	126	89	44
14	30	45	51	45	30	14
4	10	16	19	16	10	4
1	3	6	7	6	3	1
	1	2	3	2	1	
		1	1	1		
			1			

-

1.

.

$$44 + 89 + 126 + 141 + 126 + 89 + 44 = 659$$

46.

5

?

5

1

1	1	1	1	1
2	2	2	2	1
2	4	4	4	3
6	8	8	7	3
6	14	16	15	10

6, 14, 16, 15 10

61,

47.

$xyz$

$3 \times 1$

$x \leq 3, y \leq 3, z \leq 3$  (

$x, y, z$

).

$xyz$

$zyx$ .

)

?

)

,

$abc$

$bcd$  (

$a, b, c, d$

).

. )

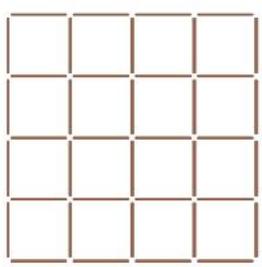
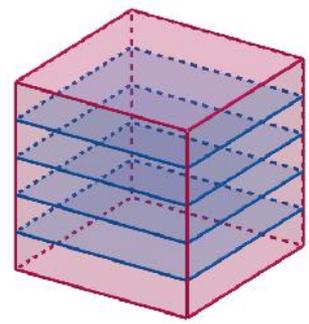
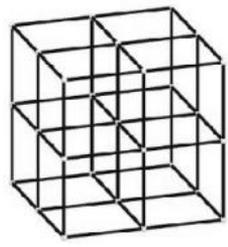
$xyx$

$4 \cdot 4 = 16$ .

$xyz \quad x \neq z \quad \frac{4 \cdot 4 \cdot 3}{2} = 24 \quad ( \quad 2$   
 $xyz \quad zyx \quad \cdot \quad ,$   
 $16 + 24 = 40 \quad \cdot$   
 $) \quad 001, 002, 003 \quad ,$   
 $113, \quad 220, 221, 223 \quad 110, 112,$   
 $330, 331, 332.$   
 $12$   
 $\cdot \quad 38 \quad ,$   
 $: 000, 001, 010, 101, 011, 111, 112, 121, 212, 120, 202, 020, 200,$   
 $003, 030, 303, 032, 323, 232, 320, 201, 013, 131, 313, 130, 301, 012, 122,$   
 $222, 220, 203, 033, 333, 332, 321, 213, 132, 322.$

48.

$1 \times 1 \times 1 \quad 1$   
 $2 \times 2 \times 2 \quad 12$   
 $( \quad ) \quad 42$   
 $8 \times 8 \times 8$   
 $9 \times 9 \times 9.$   
 $n \times n \times n$   
 $( \quad )$   
 $n+1$   
 $n \times n \quad ( \quad )$



$n+1$   
 $n \quad n+1$   
 $n$   
 $2n(n+1)$   
 $n+1$   
 $2n(n+1)^2$

$n$  . ,

$$(n+1)^2 - n(n+1)^2$$

$$n \times n \times n \quad 3n(n+1)^2$$

$8 \times 8 \times 8$

$$9 \times 9 \times 9 \quad :$$

$$3 \cdot 9 \cdot (9+1)^2 - 3 \cdot 8 \cdot (8+1)^2 = 3 \cdot 9 \cdot 10^2 - 3 \cdot 8 \cdot 9^2 = 3 \cdot 9 \cdot (10^2 - 8 \cdot 9) = 756$$

49. 10 ,

?

$$1 \quad 10.$$

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!.$$

2, 3, 4, 5, 6, 7, 8, 9, 10 , 2, 3,

4, 5, 6, 7, 8, 9, 10, 1. ,

3, 4, 5, 6, 7, 8, 9, 10, 1, 2. -

10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 7 ,

9 . ,

10 , ...

$$\frac{10!}{10} = \frac{10 \cdot 9!}{10} = 9! = 362880.$$

.  $n$

$$\frac{n!}{n} = (n-1)!.$$

50. (0,0) -

1 .



(6,14)?

2), (0,0) (3,7). 10

$$\frac{10 \cdot 98}{3 \cdot 2 \cdot 1} = 120.$$

51.

	2	1	2		

( ) ?  
 A, B, C, D, E  
 $A + B = 2, B + C + D = 1, D + E = 2.$   
 $(A, B, C, D, E) = (2, 0, 1, 0, 2), (2, 0, 0, 1, 1)$

(1, 1, 0, 0, 2).

5, 3, 2, 3 2

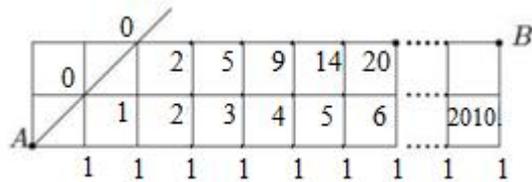
$$\frac{5 \cdot 4}{2} \cdot 2 \cdot 1 + \frac{5 \cdot 4}{2} \cdot 3 \cdot 2 + 5 \cdot 3 \cdot 1 = 95.$$

52.

$$2 \times 2011$$



( )  
 A, B, l, A, B, 2011, 2012



1,

0, 1, 2, 3, 4, 5, 6, ..., 2010.

$$2 = 2 + 0 = 2 + 1 - 1,$$

$$5 = 3 + 2 = 3 + 2 + 1 - 1,$$

$$9 = 4 + 5 = 4 + 3 + 2 + 1 - 1,$$

$$14 = 5 + 9 = 5 + 4 + 3 + 2 + 1 - 1,$$

$$20 = 6 + 14 = 6 + 5 + 4 + 3 + 2 + 1 - 1.$$

$$(k+2) -$$

$k$

1,

$a_{k+2}$

$$(k+2) -$$

$$a_{k+2} = k + (k-1) + \dots + 2 + 1 - 1 = \frac{k(k+1)}{2} - 1. \quad (1)$$

(1)

$k=1,2,3,4,5,6.$

$k+3$

$$a_{k+3} = a_{k+2} + (k+1) = \frac{k(k+1)}{2} - 1 + (k+1) = \frac{(k+1)(k+2)}{2} - 1,$$

$k.$

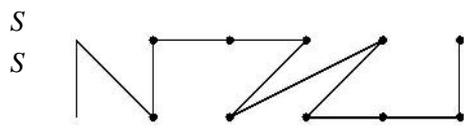
$$a_{2012} = \frac{2010 \cdot 2011}{2} - 1 = 2021054.$$

53.

(0,0)

(5,1)

$$S = \{(i, j) \mid i=1,2 \quad j=1,2,3,4,5\}.$$



(0,0) (5,1)  
 ? (  
 .)  
 .  
 (0,0), (1,0), (2,0), (3,0), (4,0) (5,0)  
 (0,1), (1,1), (2,1), (3,1), (4,1), (5,1)

(0,0), (5,1).  
 , 5 10 (0,0), (1,0), (2,0),  
 (3,0), (4,0) (5,0).  $\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$

54. 12 .  
 ,  
 ?

$A_n$   
 $n = 2m$  ,  $A_2 = 1,$   
 $A_4 = 2$   $A_{12}$  .  
 $n \geq 6,$   $a_1, a_2, \dots, a_n$  ,  $a_1$   
 $a_1$   $a_2$   $a_n$ ,  
 $A_{n-2}$  .

$a_1$   $a_{2k}, 1 < 2k < n,$   
 $A_{2k-2} \cdot A_{n-2k}$  .  
 $A_6 = 1 \cdot 1 + 2 \cdot 2 = 5,$   
 $A_8 = 2 \cdot 5 + 2 \cdot 1 \cdot 2 = 14,$   
 $A_{10} = 2 \cdot 14 + 2 \cdot 1 \cdot 5 + 1 \cdot 2 \cdot 2 = 42,$   
 $A_{12} = 2 \cdot 42 + 2 \cdot 1 \cdot 14 + 2 \cdot 2 \cdot 5 = 132.$

1, 2, 5, 14, 42 132

55. 2010  $a, b, c$

---

$n = 2k$

$A_n$

$n$

$n + 2$

$a$

$b$

$n + 2$

$aa, bb, cc, ac, ca, bc$

$cb$

$n$

$(3^n - A_n)$

$6$

$n + 2$

$ab, ba,$

$ac, ca, bc$

$cb$

$A_{n+2} = 7A_n + 6(3^n - A_n) = 2 \cdot 3^{n+1} + A_n$

$A_{n+2} = 2 \cdot (3^{n+1} + 3^{n-1} + \dots + 3) + A_0$

$A_0 = 0$

$A_n = \frac{3(3^n - 1)}{4}$

$A_{2010} = \frac{3(3^{2010} - 1)}{4}$

7.

1.

45

?

0

0,5

1

$n$

$n-1$

$$\frac{n(n-1)}{2}$$

A B B A).

$$\frac{n(n-1)}{2} = 45,$$

$$n(n-1) = 90,$$

$$n(n-1) = 10 \cdot 9,$$

$$n(n-1) = 10 \cdot (10-1),$$

$$n = 10, \dots$$

10

2.

130

?

$x+2$

$x$

$$130 - 10 = 120$$

$$\frac{x(x-1)}{2} = 120,$$

$$x(x-1) = 16 \cdot 15.$$

$$x = 16$$

$$x + 2 = 18$$

3.



, 12 , 6 .  
 ?

15 :

-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

12 , 12

3 . , 12 ,

6 , , 8 0 ,

4. 5 ,

20 , -  
2 . -

?

( 3 , .)  
2 3

6  
18 17 . -  
0

$$2 + 5 \cdot 3 + 1 = 18$$

20 . ,

2 ,

, ... 2 \cdot 3 = 6 .

5. 7 , -

53 . 3 , 0 ? ( . )  
 $\frac{7 \cdot 6}{2} = 21$   
 $3 + 0 = 3$  ,

$1 + 1 = 2$  , . . . 1 .  
 $21 \cdot 3 = 63$  . , -  
 53 ,  $63 - 53 = 10$

6. 5 , -  
 7, 6, 5, 4 3 .

( 3 , 0 ? )  
 $\frac{5 \cdot 4}{2} = 10$  . -  
 $7 + 6 + 5 + 4 + 3 = 25$  . -  
 $3 \cdot 10 - 25 = 5$  .

4 ,  
 , 3 3 .  
 , 4 ,  
 $4 + 2 = 6$  ,

4 1 1 -  
 7 2 1 -  
 , 5 1 2  
 $5 \cdot 2 = 10$   
 $10 - (1 + 1 + 2) = 6$  -

6 1 3 ,

3 3 1 3 -  
 , 3 1  
 , , , -

7. 15 -  
 30 1 , 0 -  
 -

?  
 $\frac{15 \cdot 14}{2} = 105$   
 ,  $105 - 30 = 75$   
 $3 \cdot 75 + 2 \cdot 30 = 285$   
 $285 : 5 = 57$  , 57 ,

8. 10  
 3 , 0 ,  
 1 .  
 M ,  
 m ) 98,  
 M ?  
 ) 98  
 M = 19 , m ?  
 2 . 3 ,  
 x

..... y .....

$$2x + 3y = 98.$$

$$\frac{10(10-1)}{2} = 45, \quad x + y = 45, \quad \dots \quad x = 45 - y.$$

$$2(45 - y) + 3y = 98, \quad y = 8 \quad x = 37.$$

) M 9 8

$$M = 8 \cdot 3 + 1 = 25.$$

) M = 19, A :

1) 6 , 1 2 9

A.

8

1 , 9 8

2) 1 , m = 8 . 5 4

3 , 5 33

3

A, 3 6

m = 6 m = 5, 4 5

A, 2 7

m = 7 m = 6, 3 6

A,

(

A), m = 7 m = 8.

m 5, 6, 7 8.

9. 3

3:0 3:1,

3, 2, 0, 1. 3:2

A, B, C, D.

A	9	9	1
B	5	7	6
C	2	4	8
D	2	4	9

A 9, 3:0 3:1.

A-B	3:0	3:...
A-C	3:0	3:...
A-D	3:0	3:...
B-C		
B-D		
C-D		

B 7, A. B C D, A 3:0

A-B	3:0	3:1
A-C	3:0	3:0
A-D	3:0	3:0
B-C		3:...
B-D		3:...
C-D		

D 9, C 2, D 2-1, 3:2

$A-B$	3:0	3:1
$A-C$	3:0	3:0
$A-D$	3:0	3:0
$B-C$		3:...
$B-D$		3:...
$C-D$	2-1	3:2

,  $D$  4 ,  $B$  3:2 .  
 $B$ ,  $C$  4 , 3:1 .

$A-B$	3:0	3:1
$A-C$	3:0	3:0
$A-D$	3:0	3:0
$B-C$	3-0	3:1
$B-D$	2-1	3:2
$C-D$	2-1	3:2

10.

: , .  
 $a, b, c$  ,  $a > b > c$  .  
17  
, 10 8 .  
.  $a+b+c$   
 $1+2+3=6$  .  
 $17+10+8=35$   $a+b+c \geq 6$  .  
 $a+b+c=7$  5 .  
7  
:  $7=4+2+1$  .  
17  
 $17=4+4+4+4+1$  ,

10

$$10 = 2 + 2 + 2 + 2 + 2 = 4 + 2 + 2 + 1 + 1.$$

8

$$8 = 4 + 1 + 1 + 1 + 1 = 2 + 2 + 2 + 1 + 1.$$

	4	4	4	4	1
	2	2	2	2	2
	1	1	1	1	4

	4	4	4	4	1
	2	2	1	1	4
	1	1	2	2	2

11.

30

16

10

5

0

$$16 \cdot 30 = 480$$

$x$

$$x > 240$$

$$10x + 5 \cdot \frac{480-x}{2} + 0 \cdot \frac{480-x}{2} = 1200 + \frac{15x}{2} > 1200 + \frac{15 \cdot 240}{2} = 3000.$$

---

,

$$160 + 155 + 150 + 145 + \dots + 25 + 20 + 15 = \frac{30(160+15)}{2} = 2625,$$

3000. , -

.

**8.**

1.  $A = \{5, 6, 7, \dots, 74, 75\}$   
 :  $A$  ,  $A$  , ?  
 .  $A$  (  $A$  ,  
 ). , 5 ,  
 $k = 5, 6, 7, \dots, 14$   $A$  5.  $5k$   
 6 14  
 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59,  
 61, 67, 71 73

.  $A$  -  
 . , 16.

2. 90 -  
 . 2023 -  
 ,  
 . 90  
 45 .  
 ,  
 .  
 45 · 45 = 2025 . -  
 2023 . , 2023 -  
 ,  
 , 2023 -  
 ,

3.  $n \geq 3, n \in \mathbb{N}$  .  
 $n^3$  1.

$A$  ,  $B$  ,  $C$   
 $D$

$$A = B, A = C, A = D, B = C, B = D, C = D$$

$n$ ,

$$A = 8 \quad (8-), B = 12(n-2) \quad (12-), C = 6(n-2)^2 \quad (6-), D = (n-2)^3 \quad ($$

$$A = B \quad A = C, \quad 8 \neq 12(n-2) \\ 8 \neq 6(n-2)^2. \quad A = D, \quad 8 = (n-2)^3, \quad n = 4. \\ B = C \quad 12(n-2) = 6(n-2)^2, \quad \dots n = 4. \\ B = D \quad 12(n-2) = (n-2)^3, \\ 12 = (n-2)^2, \\ C = D \quad 6(n-2)^2 = (n-2)^3, \quad \dots n = 8.$$

4.  $n \geq 3$ .  $1, 2, \dots, n$ .  
 $10$ ,  
 $n$   $n$   $($   $1$   
 $)$ .

$$\frac{n(n+1)}{2} \quad 10 \\ S, \\ \frac{n(n+1)}{2} = 10S, \quad 20 | n(n+1). \\ n \quad 10, \\ n \leq S = \frac{n(n+1)}{20}. \quad n+1 \geq 20, \quad n \geq 19. \\ n \quad 20 | n(n+1) \quad 19, 20 \quad 24 \quad ( \quad n \\ n+1 \quad 5). \quad n = 19 \quad n = 20$$



6.  $3 \times 1$   $16$   $4 \cdot 3 \cdot 2 \cdot 1 = 24$

$3 \cdot 2 \cdot 1 = 6$   $3 \times 3$   $4$  ( ? ).  $24 \cdot 6 \cdot 4 = 576$

7.  $18 \times 18$   $18 \cdot 18 = 324$

?  $a, b, c$   $a, b, c$   $a, b, c$

- 1) *abcabcab...*,
- 2) *acbcbacb...*,
- 3) *bacbacbac...*,
- 4) *bcabcabca...*,
- 5) *cabcabcab...*,
- 6) *cbacbacba...*

- 1)
- 4) 5).

5), 4), 5), 4)). *abc*,

1) 6)

$$6 \cdot 2 = 12$$

8.

5x5



15	11	37	11	15
11	21	40	21	11
37	40	26	40	37
11	21	40	21	11
15	11	37	11	15

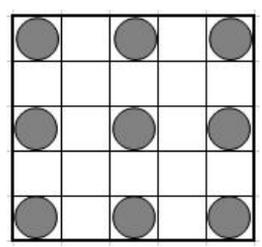
24

9

8

9

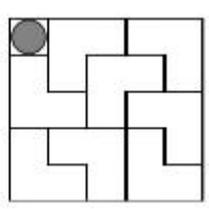
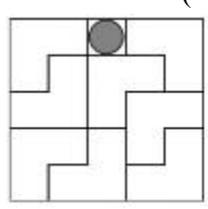
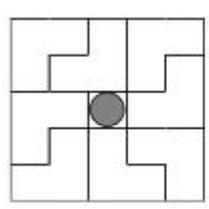
9



8

24

( )



26,

15

37.

$$26 + 4 \cdot 15 + 4 \cdot 37 = 234.$$

9.  $2 \times 2$   $7 \times 7$   $16$   $16$   $48$   $49$

10.  $10$   $10$   $1$   $k$   $k$   $k$

1, 2, 3, ..., 10, 3, ..., 10. 1, 2, 1, 3, 5, 7 1, 2, 2, 4, 6, 1, 7, 8

9.  $k=5$  1

1, 3, 5, 7, 9 2, 4, 6, 8, 10.

2 2

( 1, 3, 5, 7, 9,

2, 4, 6, 8, 10. 2, 4, 6, 8,

11.

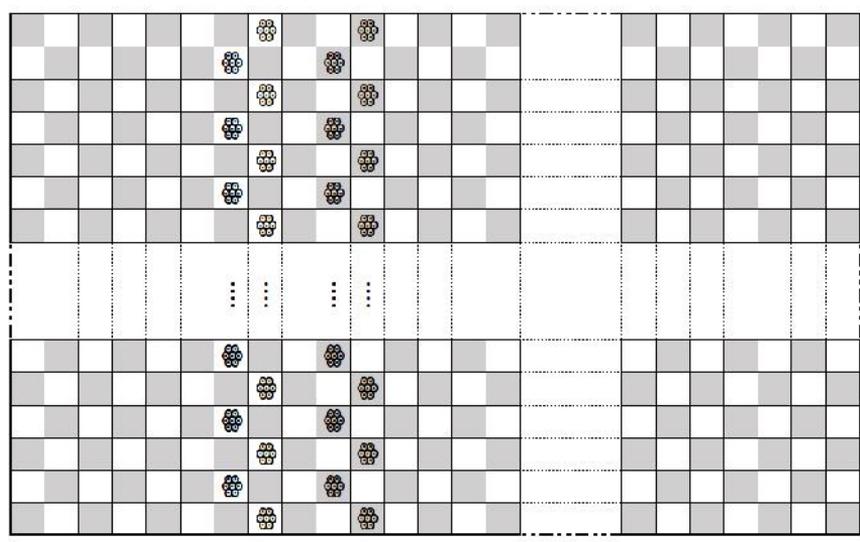
( 25 125 ), -

( ).

25 ,  $2 \cdot 25 = 50$

50

50



9.

1.

$( \quad )$ ,  
 $100$   
 $x$   
 $x+1$   
 $A$   
 $A$   
 $2, 3, 4, \dots$   
 $4$ ,  $A \leq 4x+4$ .  $100$   
 $x+1-100 = x-99$   
 $A \geq 16 \cdot 100 + 3(x-99) = 1303 + 3x$ .  
 $1303 + 3x \leq 4x + 4$ ,  $x \geq 1299$ .  
 $1299$   
 $100$   $99$   
 $12$

2.

$10$ ,  $20$ ,  $40$ ,  $80$   
 $x$   
 $( \quad )$   
 $x$   
 $?$   
 $a_1 \geq a_2 \geq a_3 \geq a_4 \geq a_5, a_4 \geq 2$ ,  
 $1, 2, 3, 4, 5$ .  
 $a_1 \leq a_2 + a_3 + a_4 + a_5$ . (1)  
 $1$ ,  
 $(1)$   
 $(1)$

$$\dots, \tag{1}$$

$$1, 2 \quad 3. \quad a_1 = a_4 > 1, \quad a_1 = a_2 = a_3 = a_4$$

$$4,$$

$$a_4 \leq (a_1 - 1) + (a_2 - 1) + (a_3 - 1) + a_5,$$

$$2a_1 + a_5 \geq 3, \quad a_1 > 1.$$

$$a_1 > a_4 \tag{1}$$

$$a_1 \leq a_2 + a_3 + a_4 + a_5 - 1,$$

$$a_1 - 1 \leq (a_2 - 1) + (a_3 - 1) + a_4 + a_5,$$

$$\tag{1} \quad 1, 2 \quad 3$$

$$a_1 = a_2,$$

$$2 \quad a_2 \leq (a_1 - 1) + a_3 + a_4 + (a_5 - 1) \Leftrightarrow a_3 + a_4 + a_5 \geq 2,$$

$$a_1 > a_2$$

$$a_1 - 1 \leq a_2 + a_3 + a_4 + (a_5 - 1),$$

$$\tag{1}.$$

$$3. \quad 16$$

$$x \leq 150$$

( ..... ) 16  
16 , ,

..... , -  
..... ,

..... 8 .

: ..... -  
, , -

..... , -  
, .

..... 8.  
, ,

..... ,  
, ,

..... , -  
4. ,

: ..... : 1, 1, 1, 1; ..... : 1, 1, 2; ..... : 2, 2;  
: 4.

4. ..... 1, 2, 3, ..., 20. -  
-

.....  
.....

..... ?  
..... ,

..... 2. 2. 1  
2. 1



100.

7.

5,

5.

?

5,

7?

. 1)

(

7).

14: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.

7

:

17, 27, 37, 47, 57, 67, 71, 72, 73, 74, 75, 76, 78, 79, 87, 97,

16

30

:

5, 10, 15, 20, 25, 30, 40, 45, 50, 55, 60, 65, 80, 85, 90, 95, 100,

..

17

5, ..

:

51, 52, 53, 54, 58, 59,

6

23

7

2)

5:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100,

20

5, ..

51, 52, 53, 54, 56, 57, 58, 59,

8

28

7:

7, 14, 21, 28, 42, 49, 63, 77, 84, 91, 98,

..

11

7, ..

17, 27, 37, 47, 67, 71, 72, 73, 74, 76, 78, 79, 87, 97

14

25

3

7.

2023×2023

(1×1

j )  
 j M, j S. :

1)

2)

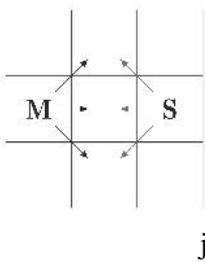
(  
 ).

3)

j j  
 j  
 (s<sub>p</sub>)  
 m<sub>p</sub> )  
 j j  
 . K j -  
 j ?

s <sub>p</sub>				m <sub>p</sub>
M				S

(1012- )  
 ( ) j . -  
 j ” j “: „ j j j j -  
 j “: „ j j j j -  
 j “: „ j j j j -  
 j j j ( -  
 ) j j j j -  
 ;  
 1) 2) ( ), -  
 j . 3) -  
 j -



8.

8 9

1, 2, 3, 4, 5, 6, 7,

3

3.

3,

3

3

3,

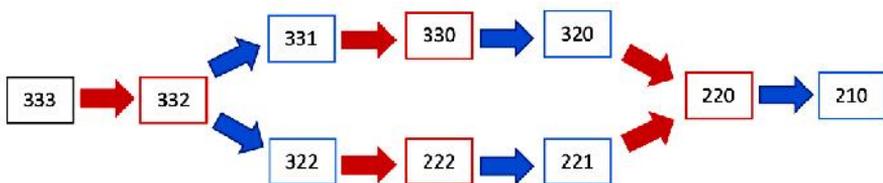
3

?

1, 2, 3, 4, 5, 6, 7, 8, 9 :  
 {1,4,7}, {2,5,8}, {3,6,9}.

3,

3.



9.

11

2013

( ).

?

$$2013 = 11 \cdot 183,$$

$$2 \cdot 183 = 366$$

$$2n + 1$$

$n$

$n$

$n = 182$   $2 \cdot 182 + 1 < 2 \cdot 183 = 366$   
 $182$   $11,$   
 $11,$   $182$   $n$   $11n$   
 $n-1$

( ) .

$n = 183$   $182$

10.

1.  $\overline{abcd}$  .

$\overline{abcd}$  :  $\overline{abcd}$

0, 1, 9,

1.  $\overline{abcd}$  -

2621 : -

) 2016, ) 2128.

1, ,

:  $(a+c)-(b+d)$ .

:  $(2+2)-(6+1)=-3$ .

) 2016  $(2+1)-(0+6)=-3$ , . . .

$(a+c)-(b+d)$  -

: 2621, 2632,

2643, 2654, 2665, 2676, 2566, 2456, 2346, 2236, 2126, 2016.

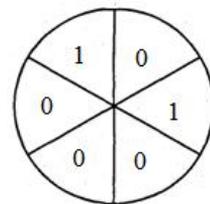
) 2128  $(2+2)-(1+8)=-5 \neq -3 = (2+2)-(1+6)$ ,

2621.

2. 6 ,

1, 0, 1, 0, 0, 0.

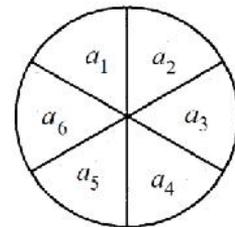
1



$a_3, a_4, a_5, a_6$ .

$a_1, a_2,$

$$A = a_1 - a_2 + a_3 - a_4 + a_5 - a_6 .$$



(

1, )  
 $A = 1 - 0 + 1 - 0 + 0 - 0 = 2,$   
 $a_1 = a_2 = a_3 = a_4 = a_5 = a_6$   
 $A = 0.$  ( )

3. 1, 2, 3, 4, 5, 2022. -  
 $a - 1$   $a + 1,$   $a$   $b,$   $b - 3$   $b + 3.$  -  
 $b$   $a$   $b - 3$   $b + 3.$  -  
: 2023, 5, 4, 3, 2, 1. ( )  
).  
 $-1 + 3 = 2$   $-1 - 3 = -4,$   $+1 + 3 = 4$   $+1 - 3 = -2$  -  
( )

4. (a, b) : I  
(a - b, b), II (a + b, b)  
III (b, a). -  
(19, 94). , -  
: ) (19, 96), ) (19, 95).  
(a, b)  
 $NZD(a, b) = NZD(a + b, b) = NZD(a - b, b) = NZD(b, a),$  -  
)  $NZD(19, 94) = 1 \neq 19 = NZD(19, 95)$

(19,94) (19,95).

) NZD(19,94) = 1 = NZD(19,96)

(19,94) (19,96).

:

$(19,94) \xrightarrow{III} (94,19) \xrightarrow{I_4} (18,19) \xrightarrow{III} (19,18) \xrightarrow{I} (1,18) \xrightarrow{III} (18,1) \xrightarrow{II} (19,1) \xrightarrow{III} (1,19) \xrightarrow{II_5} (96,19) \xrightarrow{III} (19,96).$

5. 2015 :

,  
.  
-  
1000 . 1004  
, , 1, 5 3, -  
153?  
.  
-  
, .  
+ + ,  
+ + = , ( ) + + = ,  
+ + = , + + = . , -  
, . . , , ,  
153.

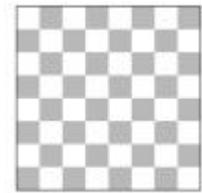
6. 25 :  $5 \times 5$  1

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

?

7.

$8 \times 8$



?

$8$   $k$  ,  $8-k$   $8$   $C$

$$C - k + (8 - k) = C + 8 - 2k .$$

$32, \dots$

8.

$10 \text{ cm}$  ,

$100$  -

$1 \text{ cm}$  .

$25$

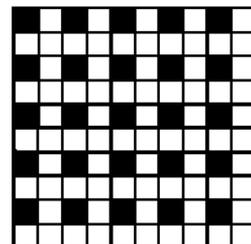
$4 \text{ cm}$

$1 \text{ cm}$  .

?

$25$

$75$





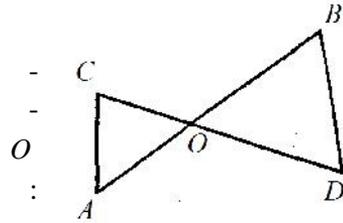
10.

$2n (n > 1)$

$n$

(  $AB \quad CD$  ).

$AC \quad BD$   
 $AB \quad CD$



$ADBC$   
 $AC \quad BD,$

( )

$AB \quad CD$

$ADBC$   
 $AC \quad BD.$

$\triangle AOC \quad \triangle BOD,$

$$\begin{aligned} \overline{AC} + \overline{BD} &< (\overline{AO} + \overline{CO}) + (\overline{OD} + \overline{DB}) \\ &= (\overline{AO} + \overline{OD}) + (\overline{CO} + \overline{OD}) \\ &= \overline{AB} + \overline{CD}. \end{aligned}$$

11.

---

( . ) .

.

, . A -

. A , ( -

) , ( -

, ( )

( ) , .

, ,

. ,

, ,

, .

**11.**

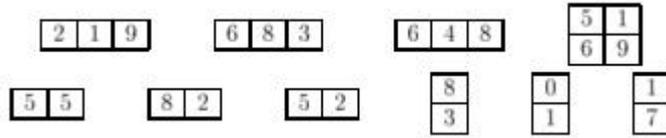
1.  $30$  . ,  
 ,  
 . ?  
 .  
 ,  $30 : 3 = 10$  .  
 ,  
 2, 5, 8, 11, 14, 17, 20, 23, 26 29,  
 .  
 .  
 32 , 10  
 3, 6, 9, 12,  
 15, 18, 21, 24, 27 30.

2. , -  
 . :  
 - .  
 - .  
 - .  
 .  
 600 ,  
 ?  
 . ... 99  
 99 ,  
 $99 \cdot 6 + 7 = 601$  .  
 98 .  
 -  
 : , ( , -  
 ). ... , -  
 . ,  
 . ,  
 . , 98 ,

$$98 \cdot 5 + 7 = 497$$

600 . 595,

3.

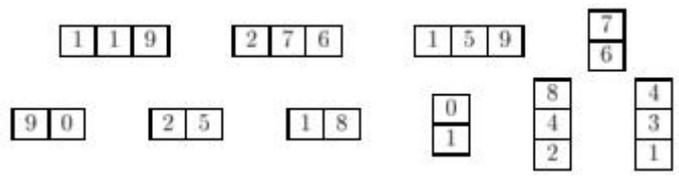


( - )  
 ( - ),  
 ) ( ) .  
 . 25 , 5.  
 ( ) , ( - )  
 ), 2, 6, 0, 4 7.  
 4(  
 ), 7(  
 ) 0 ( 0 1, 17,  
 0 1, 2,  
 219 ≠ 217. , 219, 6.  
 , 648, 683.  
 8, 3,  
 83.  
 648) 7 ( 4 ( -  
 17),

0 2. 219, 17, 7 0, 01. 1, 4 17. 4 648. - 2 82, 219 55 52 60742.

6	8	3	5	2
8	0	1	5	1
3	1	7	6	9
5	5	6	4	8
2	1	9	8	2

4.



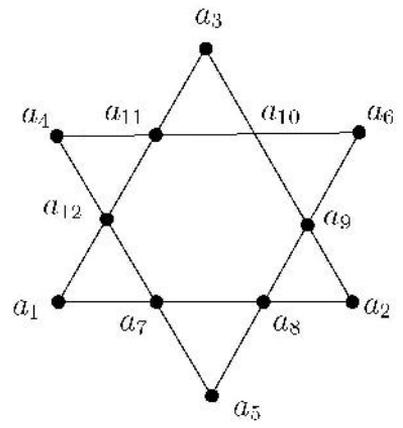
( - ) ( - ), ( - ) ( - ) . 25 , 5. ( ) ( ), 2, 3, 9 431 3 431 ,

4, 842  
 21,  
 25 276.  
 25,  
 5,  
 276,  
 76. ,  
 2, 8 9. ,  
 18 25 2 8  
 , 18  
 25 25  
 159  
 18. , 25  
 159,  
 18,  
 119,  
 01. ,  
 83298.  
 -

8	4	2	7	6
4	3	1	1	9
2	1	2	5	0
7	1	5	9	1
6	9	0	1	8

5.  $a_1, a_2, \dots, a_{12}$

1 12 -  
 . -  
 ,  
 -  
 $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$ .  
 .  
 -  
 $S$ .  
 -



$$6S = 2(1 + 2 + 3 + \dots + 12) = 156,$$

$$S = 26.$$

$$U = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 \quad V = a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12}.$$

$U$ .

$a_1 a_2 a_3$  (  $a_4 a_5 a_6$  ) -

$3S = 2(a_1 + a_2 + a_3) + V$  (  $3S = 2(a_4 + a_5 + a_6) + V$  ).

$a_1 + a_2 + a_3 = a_4 + a_5 + a_6$ ,

$U = 2(a_1 + a_2 + a_3)$ , . . .  $U$

$U = 1 + 2 + 3 + 4 + 5 + 6 = 21$   $U$  ,

$U \geq 22$ .  $U = 22$ ,  $a_1, a_2, \dots, a_6$  1, 2, 3, 4, 5, 7

$a_1 + a_2 + a_3 = a_4 + a_5 + a_6 = 11$ .

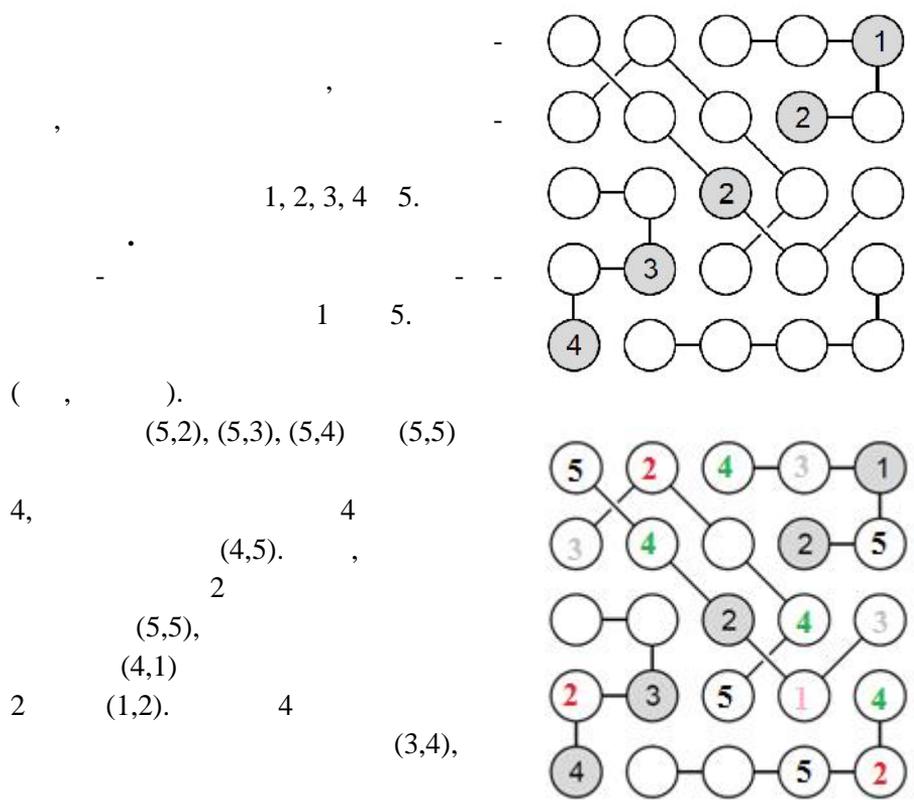
$a_1 = 1, a_2 = 3, a_3 = 7$   $a_4, a_5, a_6$  2, 4, 5 .

,  $U \geq 24$   $U = 24$  :

$a_1 = 7, a_2 = 2, a_3 = 3, a_4 = 5, a_5 = 1, a_6 = 6,$

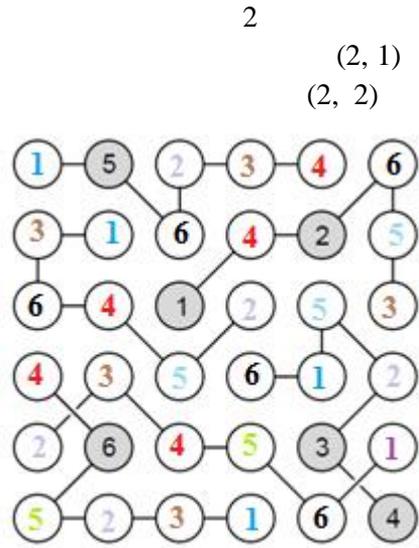
$a_7 = 8, a_8 = 9, a_9 = 10, a_{10} = 11, a_{11} = 4, a_{12} = 12.$

6.



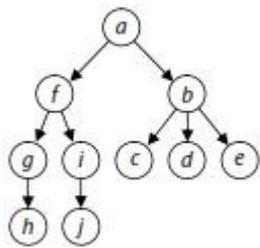
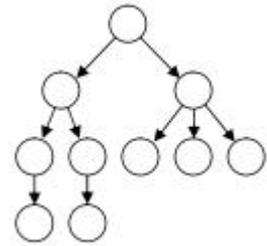


(6, 2) 2.  
 (1, 3), 6 (3, 2) 3 (6, 3).  
 3,  
 (3, 1) 6,  
 (3, 4) 2. ,  
 (3, 6) 3,  
 (1, 6) 3 -  
 6.  
 4). , (1, 1)  
 1, (5, 1) 2,  
 (6, 1) 5, -  
 (6, 4) 1 (5, 4)  
 5.



8.  
 1, 2, 3, ..., 10

(  
 ),



*a*  
 (10). -

*b, c, d, e*

$$\frac{987 \cdot 6}{43 \cdot 21} = 126$$

*f, g, h, i, j.*

*b*

*c, d, e*

6  
*f, g, h, i, j*

*f*

*g, h, i, j*

*g h*

6 ,

$i \quad j.$  -

$g \quad h$  -

$g, \quad i \quad j$  -

$i.$  , -

$126 \cdot 6 \cdot 6 = 4536.$

9.  $3 \times 3$   
9 ( )

?

45, ,

$45 : 3 = 15.$

9,

6,

9, 5, 1 9, 4, 2.

8

7, 8, 6, 1 8, 5, 2

8, 4, 3.

, 9, 5, 1, 8,

4, 3, 7, 6, 2, 9, 4, 2,

8, 6, 1, 7, 5, 3.

,  
: {9, 5, 1}, {8, 4, 3}, {7, 6, 2} {9, 4, 2}, {8,  
6, 1}, {7, 5, 3}.

$6 \cdot 6 \cdot 6 = 216$  ,

$3 \cdot 2 \cdot 1 = 6$  ,

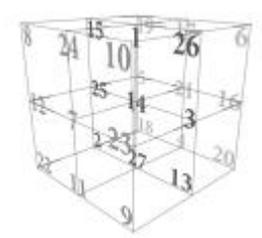
$216 \cdot 6 = 1296$

$2 \cdot 1296 = 2592.$

10. 2013 ,

$d(n) \geq 2$       2013       $d(n) = 2,$   
 $d(n) = 3,$   
 $d(n+1) = 2,$   
 $\dots$        $k$   
 $k-2$        $k-2 \geq 3$   
 $d(n) - 2$        $3 < d(n) < 2013,$        $d(n+1) =$   
 $d(1004) \leq 4, d(1005) = 2,$   
 $1005, 1006$        $1007$        $(0, 1, 2, 3, \dots,$   
 $\dots, 2$        $1007$        $(2012, 2010, 2008,$   
 $2$        $2$        $).$

11.       $3 \times 3 \times 3,$   
 $1$        $27$   
 $?$   
 $1 + 2 + 3 + \dots + 27 = 378.$   
 $27$        $3$   
 $9$   
 $378 : 9 = 42.$   
 $42$



**12.**

1.

12 cm ,  
 15 cm .  
 ?  
 , 12 15 . ,  
 NZS(12,15) = 60 cm . , 60 : 12 = 5  
 5 · 5 = 25 , 60 : 15 = 4  
 4 · 4 = 16 . ,  
 25 + 16 = 41 .

2.

1,  $\frac{1}{2}, \frac{1}{3}$   
 $\frac{1}{4}$ ,  
 ?  
 :  
 $2 \cdot 2 \cdot 2 = 8$   
 :  
 $(1 + \frac{1}{2}) \cdot (1 + \frac{1}{3}) \cdot (1 + \frac{1}{4}) = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = \frac{5}{2}$ .

3.

23 15 24 1  
 ?  
 $a < b < c < d < e$  . 15 24 10  
 10,  
 15 24

$$\begin{aligned}
 & d+e & c+e & a+b & a+c, \\
 & & & & , a+b=15, \\
 a+c=16, d+e=24, c+e=23, & & & & c=b+1 \\
 d=c+1=b+2. & & & & 15-b, b, b+1, b+2, 22-b.
 \end{aligned}$$

$$(15-b)+b=15, (15-b)+(b+1)=16 \quad (15-b)+(b+2)=17.$$

$$(15-b)+(22-b)=37-2b \quad b+(b+1)=2b+1,$$

$$18,$$

7, 8, 9, 10, 11 13 6

4.

( )

„broj“	1	2	3	4	5	6	7	8	9	0
○ ●	● ○	● ○	● ●	● ●	● ○	● ●	● ●	● ○	○ ●	○ ●
○ ●	○ ○	● ○	○ ○	○ ●	○ ●	● ●	● ●	● ●	● ●	● ●
● ●	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○	○ ○

$$11 = \begin{array}{ccc} \circ & \bullet & \bullet \\ \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}$$

$$478 = \begin{array}{cccc} \circ & \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

13 -

1,

2, 3, 5 9,

4, 6, 8 0,

7.

13

$$13-4=9$$

9

4

$$1+4+4=9, \quad 2+3+4=9, \quad 3+3+3=9.$$

771,

987

13

888. ,

987.

5.

27,

22.

?

$\overline{xyz}$ .

$$xy + z = 27 \quad x + yz = 22.$$

$$xy + z - x - yz = 5,$$

$$(x - z)(y - 1) = 5.$$

$$x - z = 5, y - 1 = 1$$

$$x - z = 1, y - 1 = 5.$$

$$y = 2 \quad x = z + 5$$

$$xy + z = 27$$

$$2(z + 5) + z = 27, \dots 3z = 17,$$

$$y = 6, x = z + 1$$

$$xy + z = 27$$

$$6(z + 1) + z = 27,$$

$$z = 3,$$

$$x = 4.$$

463

$$4 \cdot 6 + 3 = 27$$

$$4 + 6 \cdot 3 = 22.$$

6.

$n$

$n$

$n$

$n - 1$

$$n^3 - 3n + 2 = n^3 - n^2 + n^2 - n - 2n + 2 = n^2(n - 1) + n(n - 1) - 2(n - 1)$$

$$= (n - 1)(n^2 + n - 2) = (n - 1)(n^2 - n + 2n - 2)$$

$$= (n - 1)(n(n - 1) + 2(n - 1)) = (n - 1)^2(n + 2)$$

$$(n - 1)^2$$

$n + 2$

$$31^2 = 961 < 1000, \quad 32^2 = 1024$$

1024,

$$n + 2 = 1024, \dots$$

$$n = 1022.$$

7.

15

$A$   
 $B$

?

$$A - 15 = B.$$

$$A + 16 = B \quad 2A = 16 + B.$$

$$2A = 16 + A + 16,$$

$$A = 32, \quad B = 32 + 16 = 48$$

$$A + B + 17 = 32 + 48 + 17 = 97$$

8.

$$n < 2013,$$

$n$ .

$k$

$n - k$

$$\frac{n(n-1)}{2},$$

$$k(n-k), \quad \frac{1}{2} \cdot \frac{n(n-1)}{2} = k(n-k), \quad n = (n-2k)^2,$$

2023. -

$$n = 44^2 = 1936 = (1936 - 2 \cdot 946)^2,$$

946 ,

$$1936 - 946 = 990 .$$

9.

14.

?

$a, b, c$  .

$$ua + vb + wc = n,$$

$u, v, w \in \{-1, 0, 1\}$  .

14

3, 9. :

$$\begin{aligned} 1 &= 1 \cdot 1 + 0 \cdot 3 + 0 \cdot 9, \\ 2 &= -1 \cdot 1 + 1 \cdot 3 + 0 \cdot 9, \\ 3 &= 0 \cdot 1 + 1 \cdot 3 + 0 \cdot 9, \\ 4 &= 1 \cdot 1 + 1 \cdot 3 + 0 \cdot 9, \\ 5 &= -1 \cdot 1 - 1 \cdot 3 + 1 \cdot 9, \\ 6 &= 0 \cdot 1 - 1 \cdot 3 + 1 \cdot 9, \\ 7 &= 1 \cdot 1 - 1 \cdot 3 + 1 \cdot 9, \\ 8 &= -1 \cdot 1 + 0 \cdot 3 + 1 \cdot 9, \\ 9 &= 0 \cdot 1 + 0 \cdot 3 + 1 \cdot 9, \\ 10 &= 1 \cdot 1 + 0 \cdot 3 + 1 \cdot 9, \\ 11 &= -1 \cdot 1 + 1 \cdot 3 + 1 \cdot 9, \\ 12 &= 0 \cdot 1 + 1 \cdot 1 + 1 \cdot 9, \\ 13 &= 1 \cdot 1 + 1 \cdot 3 + 1 \cdot 9. \end{aligned}$$

1, 3 9.

10.

$A, B$

$A, B$

1 15

,  $B$

$A?$



1)  $(A, B)$

2)  $(C)$ .

$B, C$ ,  $A$   $B$ .

$C$ ,  $A, B, C$   $A$

13. 21, 10, 20, 21,  $Ox$ ,  $I$ ,  $1, 2, 3, \dots, 21$ ,  $1, 2, 3, \dots, 21$

11, 12, 13, ..., 21.

11 21

$I = [a_m, b_m]$ .  $[a_i, b_i]$

$a_i$   $[a_i, b_i]$   $I$ .

$I$ ,  $[a_i, b_i]$

1, 2, 3, ..., 10 11, . . .  $[a_i, b_i]$

$b_i$   $[a_i, b_i]$

I, [a\_i, b\_i]  
 11, 12, 13, 14, ..., 21 (  
 I), ... [a\_i, b\_i]

I.

14. 44 44 .  
 .  
 , ? (  
 , .)  
 . 86  
 . ” “ , -  
 . 44 43 ,  
 . 85 .  
 , 88 . ( -  
 , ) . ,  
 ( )  
 ). , A, 1 (A1)  
 B, 1 (B1), AB  
 1 A1 B1, 2 A2 B2.  
 AB 1  
 A1 B2, 2 A2 B1. 85

0 1. X, Y, Z,  
 $X_1 \geq X_2, Y_1 \geq Y_2, Z_1 = Z_2$  (  
 ?).  
 $X_1 + Y_2 + Z_1, X_1 + Y_2 + Z_2,$  -  
 $X_1 + Y_2 + Z_2, X_1 + Y_2 + Z_1$

15.

41 . -

: -



41

( ), ,

42

$$42:2=21$$

$$42:3=14$$

$$42:7=6$$

?

1.

42 2, 3 7.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{42} = 1$$

42

42,

$$21+14+6+1=42,$$

16.

0	1	2	3	4	5	6	7	8	9
9	0	1	2	3	4	5	6	7	8
8	9	0	1	2	3	4	5	6	7
7	8	9	0	1	2	3	4	5	6
6	7	8	9	0	1	2	3	4	5
5	6	7	8	9	0	1	2	3	4
4	5	6	7	8	9	0	1	2	3
3	4	5	6	7	8	9	0	1	2
2	3	4	5	6	7	8	9	0	1
1	2	3	4	5	6	7	8	9	0

10  
10  
10

$$(0+1+2+3+4+5+6+7+8+9)+(0+9+8+7+6+5+4+3+2+1)=90,$$

$$0+1+2+3+4+5+6+7+8+9=45,$$

17.

8×8

11, 12, 13, 14, 15, 16, 17 18,  
21, 22, 23, 24, 25, 26, 27  
81, 82, 83, 84, 85,

28,  
86, 87 88. 8

81	82	83	84	85	86	87	88
71	72	73	74	75	76	77	78
61	62	63	64	65	66	67	68
51	52	53	54	55	56	57	58
41	42	43	44	45	46	47	48
31	32	33	34	35	36	37	38
21	22	23	24	25	26	27	28
11	12	13	14	15	16	17	18

8, 1, 2, 3, 4, 5, 6, 7  
1, 2, 3, 4, 5, 6, 7 8.  
 $(i, j), i=1, 2, 3, 4, 5, 6, 7, 8; j=1, 2, 3, 4, 5, 6, 7, 8.$   
 $(i, j) \quad 10i + j.$

8



$$(i_1, j_1), (i_2, j_2), \dots, (i_8, j_8),$$

$$i_1 + i_2 + i_3 + \dots + i_8 = 1 + 2 + \dots + 7 + 8 = 36,$$

$$j_1 + j_2 + j_3 + \dots + j_8 = 1 + 2 + \dots + 7 + 8 = 36.$$

$$\begin{aligned} (10i_1 + j_1) + (10i_2 + j_2) + \dots + (10i_8 + j_8) \\ = 10(i_1 + i_2 + \dots + i_8) + (j_1 + j_2 + \dots + j_8) \\ = 10 \cdot 36 + 36 \\ = 360 + 36 \\ = 396. \end{aligned}$$

396, . . . -

18.

8 × 8

1, 3, 5, 7, 9, 11, 13, 15;  
17, 19, 21, 23, 25, 27, 29, 31  
1, 19 3 . ( );  
33, 35, 37, 39, 41, 43, 45, 47 .

17

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55				

8

?

7,

0, 1, 2, 3, 4, 5, 6

0, 1, 2, 3, 4, 5, 6 7.

$i - j - \dots$

$(i, j), i = 0, 1, 2, 3, 4, 5, 6, 7; j = 0, 1, 2, 3, 4, 5, 6, 7.$

3, 4, 5, 6, 7, 8, 9, 10, ...

0, 1, 2,  
 $(i, j)$

$8i + j$  ( $8i$   
 $j$ ).

1, 3, 5, 7, 9, 11, 13, ...,  $k$   
 $2k + 1$ ,  $(i, j)$

$$2(8i + j) + 1 = 16i + 2j + 1.$$

$$(i_1, j_1), (i_2, j_2), \dots, (i_8, j_8),$$

$$i_1 + i_2 + i_3 + \dots + i_8 = 0 + 1 + 2 + \dots + 7 = 28,$$

$$j_1 + j_2 + j_3 + \dots + j_8 = 0 + 1 + 2 + \dots + 7 = 28.$$

$$\begin{aligned} (16i_1 + 2j_1 + 1) + (16i_2 + 2j_2 + 1) + \dots + (16i_8 + 2j_8 + 1) &= \\ &= 16(i_1 + i_2 + i_3 + \dots + i_8) + 2(j_1 + j_2 + j_3 + \dots + j_8) + 8 \\ &= 16 \cdot 28 + 2 \cdot 28 + 8 \\ &= 18 \cdot 28 + 8 \\ &= 504 + 8 \\ &= 512. \end{aligned}$$

512, . . .

19.

$$\frac{a}{b}.$$

$a$   $b$

1

1.

$n$

$a_i$

$$b_i, a_i \geq b_i, i = 1, 2, \dots, n.$$

$$b_i \leq 1, \quad \frac{a_i}{b_i^2} \geq b_i,$$

$$\frac{a_i}{b_i} \geq a_i b_i, \quad i = 1, 2, \dots, n.$$

$$\sum_{i=1}^n \frac{a_i}{b_i} \geq \sum_{i=1}^n a_i b_i = 1,$$

20.

$n$

$a_1, a_2, \dots, a_n$

$$i = 1, 2, \dots, n$$

$$a_i \leq a_{i+1} \leq 2a_i.$$

$a_1 = 1$  ?

$$S = \sum_{i=1}^n \pm a_i \quad (1)$$

$$-a_1 \leq S \leq a_1 \quad (!).$$

$$A = \sum_{i=1}^n a_i, \quad A + S$$

$$S, \quad A + S$$

$$S = A + S - A, \quad -a_1 \leq S \leq a_1$$

$$S = 0. \quad (1),$$

21. -

$$m.$$

$$3.$$

$$m.$$

$$A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9 \quad (10). \quad -$$

$$i \quad A_i A_{i+1} A_{i+2} \quad A_{i+1} A_{i+2} A_{i+3} \quad -$$

$$3, \quad A_i \equiv A_{i+3} \pmod{3},$$

$$A_{i+6}, A_{i+9}, \dots \quad 10$$

$$3,$$

$$3. \quad 1, 4, 7, 10, 13, 16, 19, 22,$$

$$25 \quad 28.$$

$$3.$$

$$, m = 28.$$

22. -

$$n - \quad 2012.$$

$$3.$$

$$n.$$

$$A_1, A_2, \dots, A_n \quad (n). \quad i \quad -$$

$$A_i A_{i+1} A_{i+2} \quad A_{i+1} A_{i+2} A_{i+3},$$

$$3, \quad A_i \equiv A_{i+3} \pmod{3},$$

$$A_{i+6}, A_{i+9}, \dots$$



(0,1) . , 1 ( -

1, . ,

25.  $n$  . ( )

$n$ .

$n \geq 4$ .

$$n \leq 1 + 3(1 + 2) = 10.$$

$3n$

( ,  $n$  ),  $n$

4, 6, 8 10.

$n = 2k$ ,  $k = 2, 3, 4, 5$ .

1  $2k$   $i$

$i - 1, i + 1, i + k$  ( -

$n$ , ). -

$i$

$i - 2, i + 2, i + k - 1$   $i + k + 1$ ,  $k \leq 4$

$k = 5$

$A_1, A_2, A_3, A_4, A_5, B_1, B_2,$

$B_3, B_4, B_5$   $i$

$A_i$

$A_{i-1}, A_{i+1}, B_i,$   $B_i$

$B_{i-2}, B_{i+2}, A_i$  ( -

5, ).

$A_i$

$A_{i-2}$   $B_{i-1}$

$A_{i-1},$

$A_{i+2}$

$B_{i+1}$

$A_{i+1}$   $B_{i-2}$   $B_{i+2}$   $B_i,$

$B_i$

,  $n$

4, 6, 8 10.



1. D. Veljan: Kombinatorika i diskretna matematika, Algoritam, Zagreb, 2001
2. P. Mladini : Nizovi zadani kvadrati ima, aritmetički nizovi ili polinomi različitog stupnja, Matka 30/120 Zagreb
3. S. Režek: Boje na šahovnici, Matka 25/97, Zagreb
4. S. Režek: Kula, Matka 25/99, Zagreb
5. S. Režek: Šahomatematika – matematičko-šahovska razonoda, Matka 24/93, Zagreb
6. S. Sruc: Tucet zadataka iz prošlosti, Matka 30/120, Zagreb
7. Ž. Br i : Kombinatorika stotinu „Matki“, Matka 25/100, Zagreb
8. Ž. Br i : Pa to se stalno ponavlja, Matka 24/93, Zagreb
9. . : – – , +,
10. . : , , 3/2015,
11. . : , +,
12. . : , +,
13. . :  $n!$ , +,
14. . , , : II
15. . , , :
16. . : , , , 2002
17. . : ? ? , 2023
18. . , . : , , 2023
19. . , . : ( ) , , 2023
20. . , . : ( ) , , 2023
21. . : , , 2022

- 22. . , . : ,
- 23. . , . : , 2021
- 24. . , . : 2, , 2022
- 25. . , . : , 2021
- 26. . ( ) , 2020 : , +,