5-th Mediterranean Mathematical Competition 2002

1. Determine all positive integers x, y such that $y \mid x^2+1$ and $x^2 \mid y^3+1$.

2. Suppose x, y, a are real numbers such that $x+y=x^3+y^3=x^5+y^5=a$. Find all possible values of a.

3. In an acute-angled triangle ABC, M and N are points on the sides AC and BC respectively, and K the midpoint of MN. The circumcircles of triangles ACN and BCM meet again at a point D. Prove that the line CD contains the circumcenter O of $\triangle ABC$ if and only if K is on the perpendicular bisector of AB.

4. If a, b, c are nonnegative real numbers with $a^2+b^2+c^2=1$, prove that

$$\frac{a}{b^2+1}+\frac{b}{c^2+1}+\frac{c}{a^2+1} \geq \frac{3}{4}\left(a\sqrt{a}+b\sqrt{b}+c\sqrt{c}\right)^2.$$