Mediterranen Mathematics Olympiad 2015

1

Let $P(x) = x^4 - x^3 - 3x^2 - x + 1$. Prove that there are infinitely many positive integers n such that $P(3^n)$ is not a prime.

2

Prove that for each triangle, there exists a vertex, such that with the two sides starting from that vertex and
each cevian starting from that vertex, is possible to construct a triangle.

3

In the Cartesian plane $\mathbb{R}^2$, each triangle contains a Mediterranean point on its sides or in its interior, even if the triangle is degenerated into a segment or a point. The Mediterranean points have the following properties:
(i) If a triangle is symmetric with respect to a line which passes through the origin $(0, 0)$, then the Mediterranean point lies on this line.
(ii) If the triangle DEF contains the triangle ABC and if the triangle ABC contains the Mediterranean points M of DEF, then M is the Mediterranean point of the triangle ABC.

Find all possible positions for the Mediterranean point of the triangle with vertices $(-3, 5)$, $(12, 5)$, $(3, 11)$.

4

In a mathematical contest, some of the competitors are friends and friendship is mutual. Prove that there is a subset M of the competitors such that each element of M has at most three friends in M and such that each competitor who is not in M, has at least four friends in M.
