6-th Mediterranean Mathematical Competition 2003

1. Prove that the equation $x^2+y^2+z^2=x+y+z+1$ has no rational solutions.

2. In a triangle ABC with $BC=CA+\frac{1}{2}AB$, point P is given on side AB such that $BP:PA=1:3$. Prove that $\angle CAP=2\angle CPA$.

3. Let a,b,c be nonnegative numbers with $a+b+c=3$. Prove the inequality

$$\frac{a}{b^2+1}+\frac{b}{c^2+1}+\frac{c}{a^2+1}\geq\frac{3}{2}.$$

4. Consider a system of infinitely many spheres made of metal, with centers at points $(a,b,c)\in\mathbb{R}^3$. We say that the system is *stable* if the temperature of each sphere equals the average temperature of the six closest spheres. Assuming that all spheres in a stable system have temperatures between 0°C and 1°C, prove that all the spheres have the same temperature.